CAS: An Introduction

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In some parts of world there is a passionate debate about whether Computer Algebra Systems (CAS) are a legitimate educational tool. In Scotland there has been considerable interest, different States in Australia have set up significantly different mathematics examinations to take account of some allowing and some not, the use of CAS systems in the exams themselves(Evans et al. 2005). Curiously, in England there is no such debate. Many teachers will not have come across CAS in their professional lives. If they have done a degree in mathematics or engineering, they almost certainly will have. CAS software is regularly used by professional mathematicians, more commonly in applied fields and certainly by engineers and scientists. MathCad, Maple and Mathematica, all CAS systems are very familiar software in professional fields and have been available and developed for many years. Research suggests that their use in University teaching is variable. Academics who use CAS in their research and development work will use it in their teaching (Lavicza 2008). Generally these will be applied mathematicians and engineers. In England, school mathematics has tended to be rooted in pure mathematics. We learn how to factorise quadratics largely for its own sake, the skill is the most important thing. Elsewhere, the problem tends to come first. However, now that the STEM (Science, Technology, Engineering, Mathematics) agenda is beginning to lead developments in what has come to be known as 'Using and Applying' or 'Functional Skills', this all looks set to change.

So, what is CAS? From the early days of computing, computer programming languages embedded commands that were capable of evaluating mathematical expressions. Many mathematical functions were built in. CAS is simply a collection of commands capable of actually doing the mathematics. Early CAS software would simply consist of additional commands for your programming language that would, for example allow you to solve and equation, simply an expression, differentiate or integrate a function. Modern CAS does the same, except now with the advantage of powerful graphical interfaces, the algebra can be correctly formatted, the data can be looked at in sophisticated forms notably with powerful graphing facilities, the interfaces are now much more user friendly. In the end, CAS provides tools for evaluating and manipulating mathematical expressions.

With such powerful tools available for mathematics it was natural that they would be made available for school use. The most common CAS system in schools was called Derive. This was widely used throughout the 1980s and 1990s and developed into a sophisticated graphical system. There are many publications documenting the use of Derive in schools (e.g. (Kutzler & Boykett 1996). However, in England there remained the worry: what is the point learning mathematics manipulation, when you have a machine that will do it for you? The residual worry that adults cannot add up because we gave them calculators when they were at school has led any debate there may be. This debate is well set out in the the 2004 book 'The Case for CAS'. (Bohm et al. 2004). In Europe and many other parts of the world, where mathematics is taught from problem solving perspective, the problem itself is the issue. Sophisticated tools used in developing a solution are appropriate and reasonable. The actually doing of the mathematics is not the evaluation. It is the setting up of the relationships, data, functions, inputs that will solve the problem. We need to move towards a problem solving perspective, where we are not simply using the problem as a vehicle to practice mathematical manipulation. This naturally mirrors the practice of applied mathematicians and engineers doing the task for real in their professional lives.

Students can struggle to sustain their thinking through a complex problem solving activity, because there are too many aspects to be proficient in. The symbolic mathematics can sometimes become a barrier to developing the problem solving. CAS can act to ease the flow of the thinking by carrying the burden of the algebra. That is not to say that students should not learn how to manipulate algebra for themselves, just that this can be saved for another day, when that is the focus of the task. Then the two strands (Pure and Applied) can be developed together, rather than wait (perhaps forever!) for a secure and confident algebraist before problem solving is allowed. Instead the two become mutually supportive.
A calculator can be seen as a threat when pupils are allowed to use it to do simple arithmetic.

However, when used as a pedagogic tool, generations of teachers have developed activity based learning opportunities to develop their pupil's numeracy. The same with CAS. We can give a student a scientific calculator, suggest some different combinations to explore and let them investigate how the calculator (let's say) add fractions together. Asking 'how does it do that' questions like this, allows students to develop their experience in structuring and organising their investigation skills. Which types should I try next. Does that support my rule or do I need to modify and develop it further? What counts as the most general example? What is my most general rule? CAS offers the logical extension to any number based opportunities. The most general addition of fractions would be something like $\frac{2}{3}+\frac{3}{5}$. What rule works for this (and hence all) cases? With CAS we can test to see if the rule works in all cases.

The factor command is quite powerful. A favourite activity is to investigate what it does to numbers. Using this to develop a method for prime factorisation is very successful. However, when this is done students are ready to explore what it does to expressions ... of many different kinds. The last two are quite dramatic. Why does one work fine and the next one not at all? Students will want to see what happened and explore which quadratic factorise.

CAS allows us to make sense of algebra. To solve an equation we have to modify the statement of the equation to find simpler statements which allow us to find the value of the unknown which satisfies the original statement. CAS has the mathematical tools to do this. Student's can experiment with different approaches to see if they make the statement more simple or more complicated. At the end they can test the outcome and the CAS system will confirm that the statement is true when x=1. Having CAS available allows students to explore ever more complicated statements and hence see the need to develop more sophisticated mathematics for simplification and hence, solution.

With CAS we can give students a tool to manipulate algebraic expressions with a wide range of mathematics allowing them to investigate how the mathematics works. The CAS environment here is a pedagogic tool. Naturally students should go back to pencil and paper and practice the skills independent of the machine. In time students will becoming secure with a wider range of algebraic skills. Then, just as with good arithmetic, the student can choose the right tool for the job: brainpower, pencil and paper, technology. CAS completes the toolkit. There now exists integrated mathematical software, which has a set of mathematical tools: numeric, graphing, geometric, statistical. This article has only dealt with CAS on it's own. CAS facilities are complementary to all of the other modes. For example, we can transform graphs, geometrically, but what effect does that have algebraically. With CAS we can investigate.

In England it has become necessary for manufacturers to develop technological tools with the CAS facilities disabled. In this way, they are acceptable for use in public examinations. The CAS machine cannot be used in exams. However, that is not it's purpose. It is a pedagogical tool to support students learning and developing their mathematics.

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