Exploring Maths at Advanced Level

Maths Activities for the HP39gII

Chris Olley

Contents

Introduction

This book is aimed at users of the HP39gII graphing calculator, who are studying for an advanced level exam in mathematics (English A Levels, Scottish Highers, French and International Baccalaureat, German Abitur, Netherlands HAVO and VWO 3/4 level and equivalent in other countries). The aim is to provide a range of activities which will help you become a confident user of the calculator while developing your skills in different mathematical topics. There is no attempt to cover the entire course, but there is a good range of topics covered. The power of this technology is in its capacity to generate lots of mathematical information very quickly, so you can get a good feeling for mathematical ideas. Mathematics needs to be explored and with your HP39gII you can get under the skin of the ideas you need to learn about. I hope that you will try out the activities in this booklet and get into the spirit of exploration that you can then use in all of the topics you need to study.

About the Author

Chris Olley was a secondary school maths teacher in a range of comprehensive schools in London and East Africa. He currently directs the secondary maths PGCE course at King's College London. He has worked with graphing calculators since they first arrived in the late 1980s and has run sessions nationally and internationally on different approaches to dynamic ICT in maths education, of which graphing calculators are an excellent example.

Further Information and Support

Please visit www.hpgraphingcalc.org and join in the discussion threads. Share your ideas and your understanding of mathematics with others.

1. Studying Advanced Level Maths with an HP39gII Graphing Calculator

The HP39gII is a very powerful calculator. It can find solutions to a vast range of mathematical problems in algebra, calculus, probability and statistics, complex numbers, matrices and much more. Press the Math key to see the menu of commands and get a feeling for how much this machine will do.

The big message for advanced level maths students, is that you can use this machine in your exams (those for which a calculator is allowed). I would assume you have bought the calculator because you know this. It is really important that you do not think that the calculator will answer the exam questions for you. Notably, machines you can use in the exams (like the HP39gII) are not allowed to manipulate algebra for you (so called Computer Algebra Systems: CAS). So, you actually have to do the steps in the process. However, the main source of difficulty in an advanced level exam is making small errors along the way. With this calculator you can quickly calculate the solution and check that you have done it correctly and be able to move on confidently. Knowing the answer first is often a very helpful way of deciding what steps to take. Also, being able to see different representations of an object, quickly, helps you decide what the solution could be. With this calculator you can draw graphs and see tables of values of different types of functions, which you can zoom in and out of, make calculations on, even do numeric integration and differentiation.
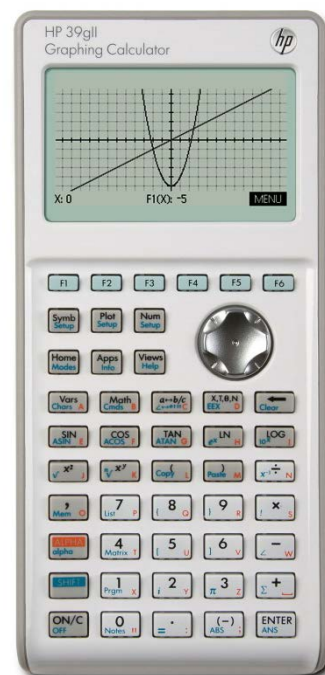
So, it won't tell you how to solve the problem, but it will get you confidently to a solution with a range of better ways of seeing. This is a fantastically powerful tool and all of the evidence from different countries where graphing calculators are used in exams says that if you can use it properly, then it will give you an advantage.

However, it's not just in the exam that the calculator is useful. While you are learning maths, it is vital that you get a good intuitive feeling about how maths works. The calculator is a fantastic tool for exploring mathematics. What happens to the graph of a quadratic $y = ax^2 + bx + c$ when you change the coefficients a, b and c? You would need to draw dozens of graphs to get a good feeling for what goes on. With this calculator you can draw as many as you like, changing the coefficients selectively. Working this way helps you get out of the other big problem with advanced level exams. Memorising methods is fine until you find a question you can't recall the method for. That normally means most of them, because they never come up just like you were expecting. Far better to understand what is going on and be able to see the mathematics from different starting points, then you can work your way through even when you can't decide which method to use. Maths is the method and you make yourself a mathematician, by exploring. The HP39gII is the tool to make that possible.

This book has a range of examples. Many of them will suggest areas of maths you can explore. But remember, they are just examples. Use the ideas to explore any new area of maths you are learning. By the time of your exam you will be so skilled in using the calculator that it will be able to support you quickly and powerfully in the exam itself.

## 2. Basic Operation

The Home Screen and the Math Menu

When you switch the calculator on, you are in the Home screen. Here you can do any type of calculation. Not just arithmetic, but using matrices, summing sequences, calculus, complex numbers and so on. Pressing the Math key shows you the menu of all of the things you can do on the Home screen. Notice that the Math menu has three main sub menus, so make sure that the MATH submenu is selected (by pressing F1). Navigate with the cursor button and note that the bottom of the menu shows you how to enter things when more than one input is needed.
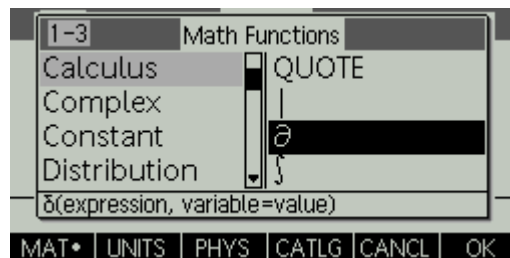
So, to do a numeric differential we find the ∂ command which tells us to enter an expression i.e. the function, then a value, i.e. the value at which we are calculating the differential, i.e. the slope.

Let's calculate the differential of x^3 at $x = 7$. We know the differential is $3x^2$, so at $x = 7$ the result should be $3 \times 7^2 = 147$.

You will have used the X,T,θ,N key to enter the X, the x^y key for the power and the = key (shift and decimal point) to set the value.
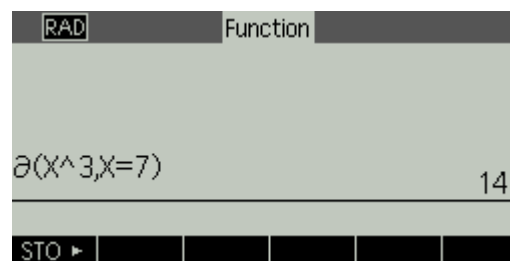
OK. Now get started. Explore!   Find any functions you already know and make them work for you. Then explore ones you've vaguely heard about.

(If you are doing Physics and/or Chemistry as well, then you will be tempted to check the UNITS and PHYS menus as well. I suggest you save this till later, but it is good to see that for scientific work, this calculator will make calculations with correct units attached. It also knows all of the key constants scientists need to know).

Working with Apps

The home screen is where you do calculations. You can always get back to the home screen by pressing the Home button. Pretty much everything else happens through Apps. Press the Apps key and scroll down the list. You will see most of your advanced level topics are covered. All of the Apps work the same way. The **Symb**, **Plot** and **Num** keys show you the three representations of all the maths you can explore with Apps. **Symb** is for the algebraic view, **Plot** shows the graph view and **Num** shows the numeric or table view. Different Apps start in different views according to the maths, for example graphing Apps start in **Symb** mode, so you can enter a function, but Statistics Apps start in **Num** mode, so you can enter the data. The bar at the top of the screen tells you which App is running at any time. If you start up a new App, all of your data from the last App you were using is saved automatically and you can come back to it any time.

You can change the way each of these representations looks using the Setup option on each key (using the SHIFT key). Explore to see what the possibilities are. Finally, you can change the way the

screen is organised using the Views menu. For example, you can split the screen and see the graph and table of values at the same time.
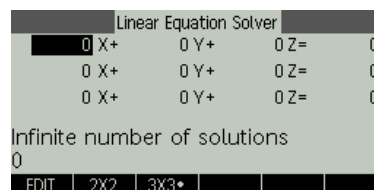
There are three main types of Apps:

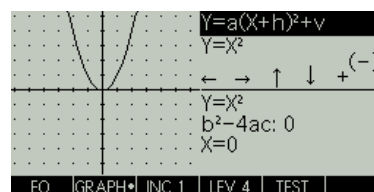1. **Function Apps**. Here you enter the data or the function without restriction.
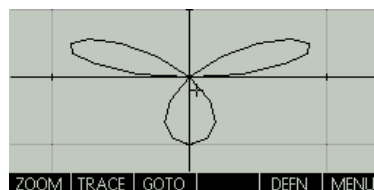
1 Polar App

2. **Solver Apps**. Here you get fast solutions to specifics types of problems.

2 Linear Equation Solver App

3. **Explorer Apps**. Here you get a pre-configured set up to make exploring specific situation easy. These also contain tests to check your understanding.

3 Quadratic Explorer App

HP39gII users are a great enthusiast community and there are additional Apps created and being created for you to download and add to the functionality of your machine. So, check regularly to see what is available, and of course, contribute yourself.

Hints and Tips

*   This calculator has detailed help built in. Press the Help button (SHIFT Views) and it will give you help on whatever you were doing.
*   You can show mathematics correctly, like this … Press Modes (SHIFT Home), Press F4 and check the Textbook display option.

*   You can save apps with your settings preserved and attach notes to remind yourself what you were doing. Press Info (SHIFT Apps) to add the notes. Then select the App in the Apps menu. Click F1 for SAVE and give it a new name. You can then share this with anyone else, or indeed online.
*   Install the connectivity software onto your computer. This way, the main operating system of the calculator can be updated when changes become available, you can upload and download Apps and you can type notes for your Apps or Programme your calculator much more efficiently with a keyboard!

3. Graphs and Functions

Linear Functions

It is very important that you are able to look at a function and have a good feel for the size, shape and position of the graph of that function. Your HP39gII is the perfect tool for exploring functions.

In the first instance you should do this for functions of the form $y = f(x)$ using the Function App. Later on you may need to do the same for polar functions, parametric functions and sequence functions, depending on the modules you are taking. There are Apps for all of these, but for now, let's focus on the Function App. Press the Apps key, choose Function, press F2 to RESET and F6 for OK to confirm, then F6 to START.

You should be very familiar with (linear) functions of the form $f(x) = mx + c$. You will know that these are straight line graphs and that they have a gradient of m and they cross the y-axis at $(0, c)$ which is called the y-intercept.

First you should make sure you know the relationship between the gradients of linear functions whose graphs are:

- Parallel
- Perpendicular

Your HP39gII is ready to receive an input for a function $F1(x)$, so try a simple Linear Function, say $F1(x) = 3x + 1$

Just type 3, press F3 for the X (or press the X,T,θ,N key or press ALPHA X), then +, then 1, then press F6 for OK.

(Notice that the F1 now has a tick next to it, which means that when we choose Plot or Num modes, F1 will show up. We can use F2 for ✓ CHK to turn functions on and off)

Try a function for F2 which will be parallel to F1. The cursor has already moved down to F2, so you can type this directly. Now press the Plot key to see if you are correct.

Press the Symb key, make sure the cursor is on the F2 line and try a new function for F2. Keep trying different functions until you are sure you know the relationship between the gradients.

Now do the same thing for Linear functions whose graphs are perpendicular.

You will need to explore different families of functions:

Polynomial functions
$$f(x) = ax + b$$
$$f(x) = ax^2 + bx + c$$
$$f(x) = ax^3 + bx^2 + cx + d$$
(etc.)

Trigonometric functions
$$f(x) = \sin(x)$$
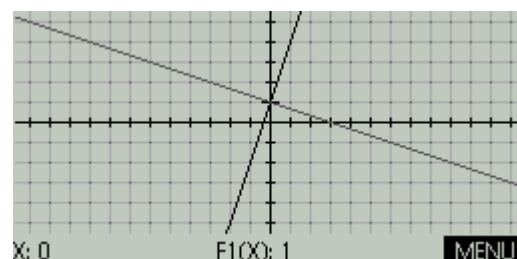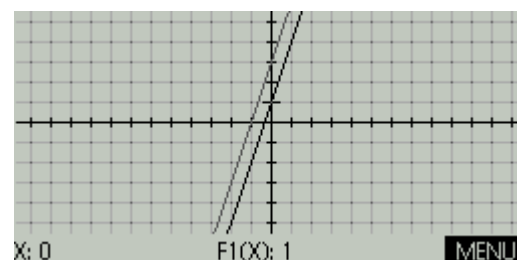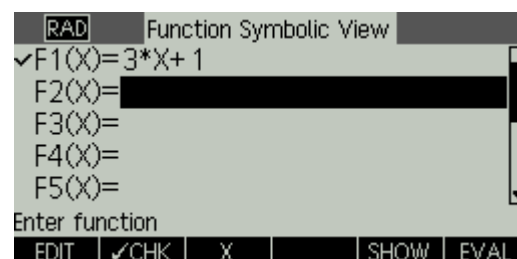$$f(x) = \cos(x)$$
$$f(x) = \tan(x)$$

Reciprocal Functions
$$f(x) = \frac{1}{x}$$

Exponential Functions
$$f(x) = a^x$$

I'm sure you knew the relationships already. But this will have helped you get used to entering and changing functions and looking at their graphs.

Quadratic Functions

Next, you should explore Quadratic functions. There are three standard ways of expressing a quadratic function:

1. Polynomial: $f(x) = ax^2 + bx + c$
2. Factorised: $f(x) = (x + a)(x + b)$
3. Completed square: $f(x) = (ax + b)^2 + c$

The first one is most familiar, but the factorised form gives the most information quickly (if the quadratic can be factorised). The completed square form probably gives the most complete picture, but the algebra involved in changing to this form is trickier. The important thing is that all three give different insights into the nature of the function. Your HP39gII will not do the algebra for you, so you will have to practice converting between these three forms with pencil and paper.

Press Apps, select Function, then RESET/OK/START and enter a function F1 in factorised form, say $F1(x) = (x - 3)(x + 5)$ press OK, then press Plot.

Press F6 for MENU, then F4 for FCN. Make sure Root is highlighted and press F6 for OK.

Now move the cursor so it is closer to the second root (clicking left a fair few times will do it). Now press MENU/FCN/Root/OK again.

The relationship between the roots and the factorised form is pretty clear. Try a few more examples to make sure.

Now get a feel for the completed square form. Replace your F1 with a function in completed square form, say $F1(x) = (x + 3)^2 - 4$ (use the x^2 key to enter the 'squared'). Now look at the position of the minimum point with MENU/FCN/Extremum/OK
Also, check the positions of the roots as before.
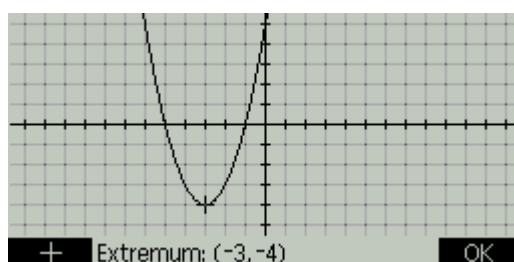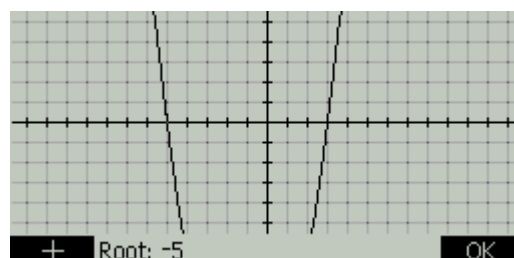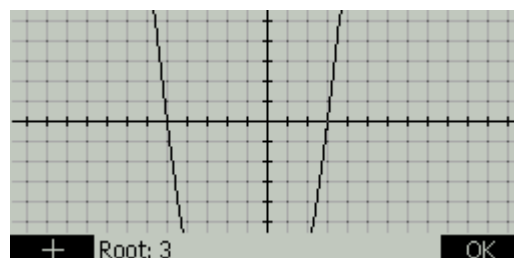
You will need to do some exploration before you can be sure of the relationship between the roots, the extremum and the values of a, b and c in $f(x) = (ax + b)^2 + c$, so keep going until you are quite sure. Then test your theory on new functions in this form.

Now you should be ready to take on the task of exploring the effect of changing a, b and c in the form $f(x) = ax^2 + bx + c$. Work on each one independently. The effect of changing the c is quite straightforward. The effect of changing the a is a bit counter-intuitive. However, just changing the b on its own is quite hard to describe. So, spend some time exploring until you are quite sure you have a good theory that you can describe accurately and simply.

Polynomials

Now you should explore other functions. Firstly, get a feel for cubic functions. Remember that different algebraic forms give different insights into the nature of the function. It is quite hard to imagine the graph of a cubic in the form $f(x) = ax^3 + bx^2 + cx + d$ but much easier in the form $f(x) = (x + a)(x + b)(x + c)$, so factorising first, if possible, is always a good thing.

Try some quartic and quintic functions as well, to get a feeling for the differences between different polynomial functions.
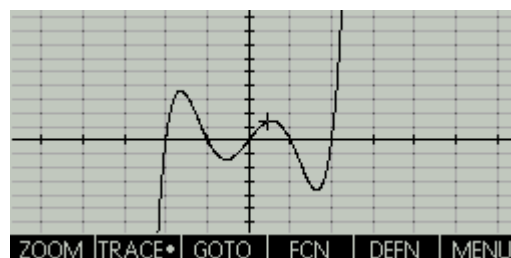
Other Functions

You should explore trigonometric, reciprocal and exponential functions in the same way.

When you have a good feel for the general shape and position of these functions, you can explore how the graphs of functions can be manipulated generally.

The best way into this is to have a good play with the graph explorer apps: Quadratic Explorer and Trig Explorer.

Press Apps and select Quadratic Explorer.
(Best to RESET then START)

Use the cursor to move the graphs up, down, left and right and spread the graph in and out with the + and − keys. Look at the effect this has on the function.

Work through the 4 levels of difficulty (LEV1 to LEV4) and TEST yourself.

Now do the same with the Trig Explorer applet.

Practice until these conclusions are clear:
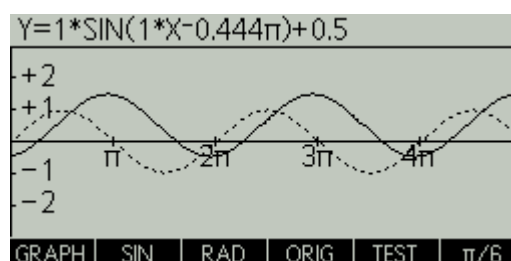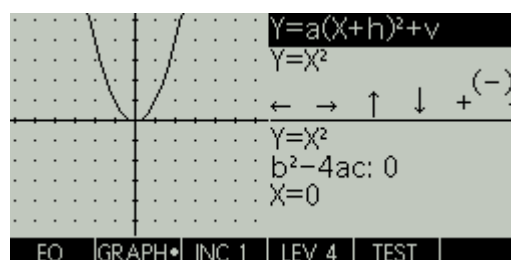
A vertical shift of a in the:

+ve direction $f(x) + a$

−ve direction $f(x) - a$

A horizontal shift of a in the:

+ve direction $f(x - a)$

−ve direction $f(x + a)$

A vertical stretch of a :

+ve (increase) $af(x)$

−ve (decrease) $\frac{1}{a}f(x)$

A horizontal stretch of :

+ve (increase) $f\left(\frac{1}{a}x\right)$

−ve (decrease) $f(ax)$

Now you should explore every different function type. Start with a reciprocal function $f(x) = \frac{1}{x}$ and ensure that you can translate and stretch it horizontally and vertically. Repeat with an exponential function $f(x) = a^x$ and use HOME/STO/ALPHA/A to change the value of A.
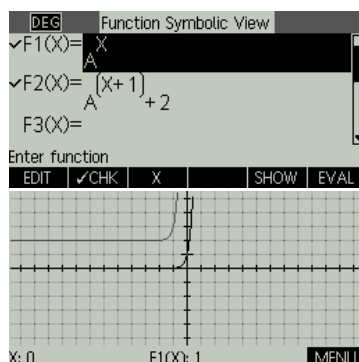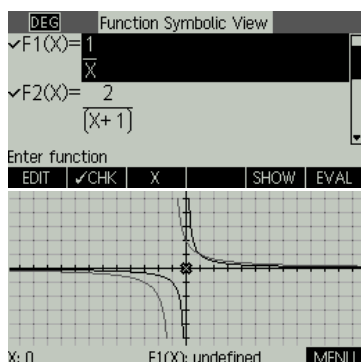
Piecewise functions

We can graph piecewise functions i.e. ones that are defined differently over different part s of the domain.

For example, we can graph a function for which $f(x) = 2x$ when $x < 1$ and $f(x) = 2$ when $x \geq 1$

We write this function as $f(x) = \begin{cases} 2x & x < 1 \\ 2 & x \geq 1 \end{cases}$

Press Apps and launch the Function applet.

(RESET/OK/START)

For F1(X) you should enter X then press SHIFT Vars (to get the Chars menu) and find the back slash / character.

Click OK.

Now enter the domain in brackets (X<2) getting the < from the Chars menu.

Click OK to see the function and the domain shown.
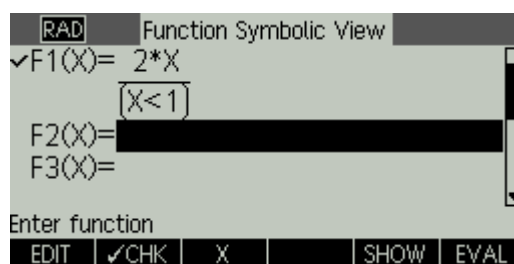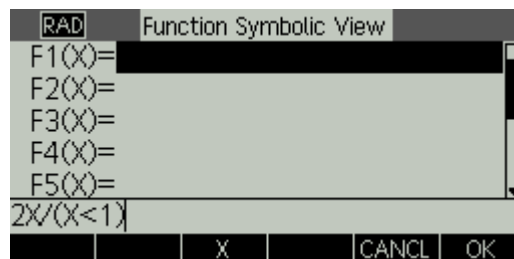
Now enter the second part of the function as F2(X)=2. You can find a $\geq$ symbol but it is easier to find $>=$ which means the same.
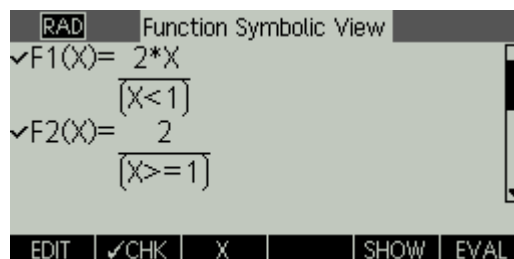
Finally press Plot to see the effect. You should explore setting up different piecewise functions, including more than two parts.
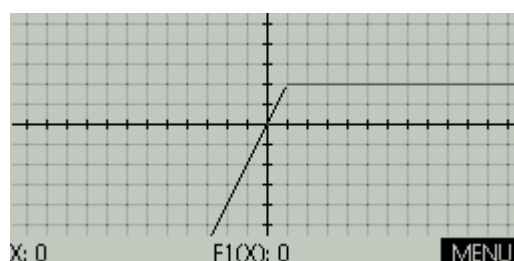
4. Matrices

Mainly you will be using matrices for Linear Algebra. The Matrix button (SHIFT + 4) is the editor. Here you can create Matrices in a convenient way. You can also type matrices directly onto the Home screen. When you have created a matrix, you should store it as a variable like M1, M2 etc.

Let's do a task. Solve the linear system:

$$2x_1 + 3x_2 - 5x_3 = 13$$
$$x_1 - 3x_2 + 8x_3 = -13$$
$$2x_1 - 2x_2 + 4x_3 = -6$$

Use the Matrix editor to enter the matrix of coefficients.

This matrix is already called M1 by the editor. Just click EDIT to get started and use the cursor keys and OK to enter each value. The Editor quickly works out the size of the matrix. When you are finished, press Home.

Alternatively, type directly [[2,3,-5],[1,-3,8],[2,-2,4]] then press F1 for STO ▶ then ALPHA M then 1 then ENTER.
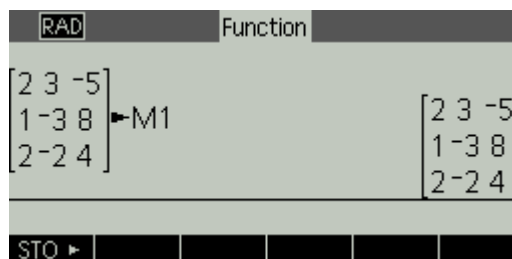
To solve the problem we pre-multiply the solution matrix, by the inverse of the coefficients matrix. So create a second matrix M2 with the right hand sides of the equations. This is a column matrix.

Use the matrix editor, or type on the Home screen:

[[13],[-13],[-6]] ▶ M2

(Note that the double square brackets are needed because each square bracket shows a new row)
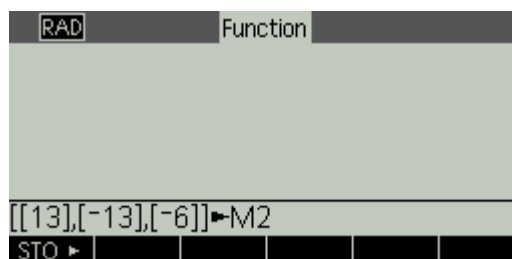
Press the Math key to find the Matrix section to see the range of operations we can apply to our matrix.
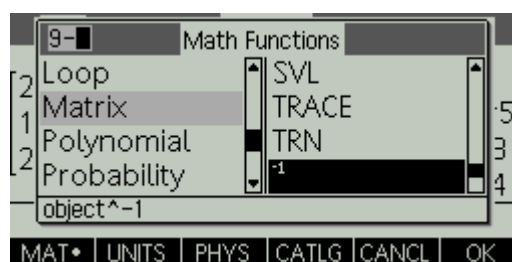
Now we are ready to find the solutions …

On the Home screen type $M1^{-1} \times M2$

(you can either use the Math/Matrix menu to get $^{-1}$ or use the x^y key and type -1).

If the Textbook display is turned off, then we can use the Show key (F5) to see the matrix correctly.

So, we have solutions: $x_1 = 1, x_2 = 2, x_3 = -1$
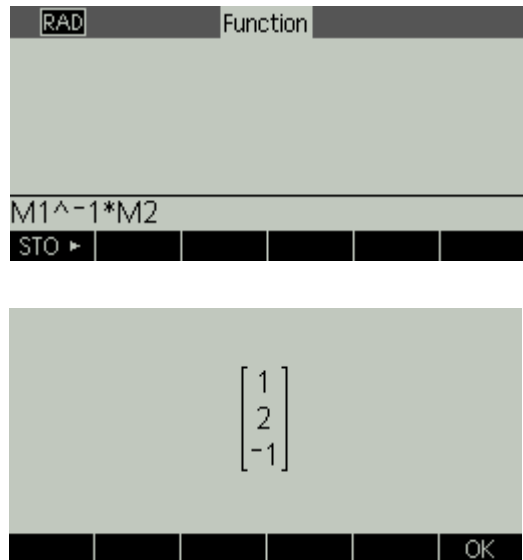
Entering the matrices can be a bit time consuming, but when they are entered you have an enormous amount that you can do with the matrices. So, now explore the other matrix functions available on the Math menu.

You can find the determinant.

You can find inverse matrices (notice that M2 has no inverse, so will generate a syntax error if you try this).

Here we have used the up cursor to highlight the matrix and SHOW to see the matrix correctly.
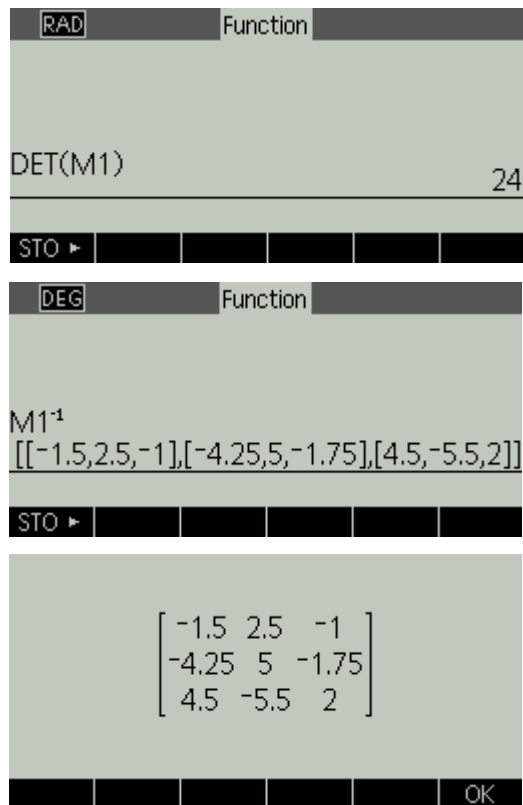
You can find Eigen Values and Vectors and do row and column operations. You should experiment to get a feel for all of the available functions and operations.

5. Sequences and Series

You will be very familiar with different continuous functions. You will have explored the relationship between the function, its graph and tables of values. With your HP39gII you can get a feel for the variation in functions which are not continuous, by exploring sequences and series.

Look at a simple sequence: 3, 5, 7, 9, 11, 13, …

The first term is 3 and the rule is that each number is 2 more than the previous number (we have $u_1 = 3$ and $u_n = u_{n-1} + 2$).

(You may know this as an arithmetic progression with $a = 1$ and $d = 2$).

Let's look at this with the sequence app.

Press Apps, select Sequence then RESET/OK/START

U1 is a sequence. We can enter its first term U1(1), its second term U1(2) and a rule for the nth term. You only need to enter what is needed. The first and second terms are not necessarily needed. In our case, we have a first term and a rule.

Enter the first term U1(1)=3

Then navigate down to U1(N). Notice that options are added for the (N-1) which we will need.

Use this to enter $u_n = u_{n-1} + 2$ as U1(N)=U1(N-1)+2

Press the Num key to see the sequence.

I pressed F4 to turn off the BIG font so you can see more of the sequence.

Use the down arrow to see even more.

Notice that using the up arrow will not allow us to go back further than the first term, because we entered a value for U1(1) saying that this is where the sequence starts.

Now press Plot to see what the sequence looks like graphically.

Press SHIFT Plot to see the plot setup options.

I have changed the plot type to Cobweb and increased the end of the Y range (YRNG) to 3 to see more of the sequence.
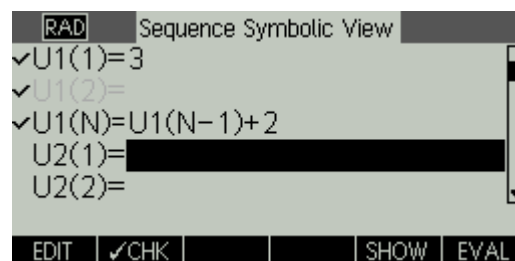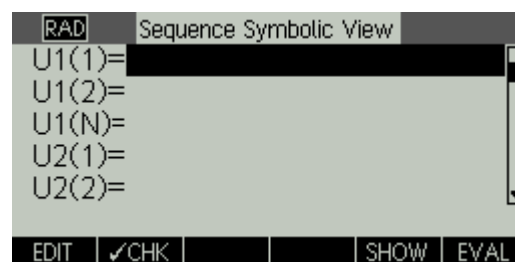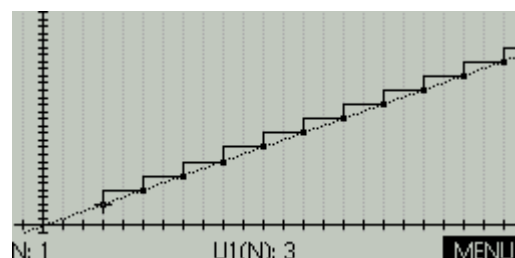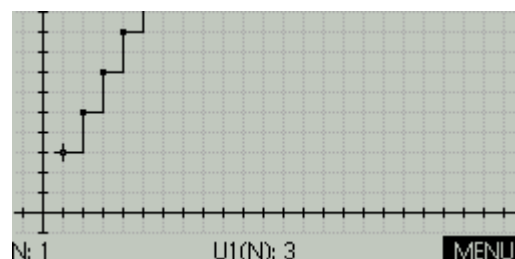
We have specified the sequence with a term-to-term rule. Now we can compare with a position to term rule, i.e. a rule for the nth term.

The first term is three and the sequence is increasing by two for each extra position so the position-to-term rule must be $u(n) = 2(n-1) + 3$. Notice it must be $n-1$ because the first term has zero 2s added on.

Press Symb and enter this rule as the second sequence U2. Enter the rule as U2(N)=2(N-1)+3

(Notice that when you do this U2(1) and U2(2) are greyed, because with a position-to-term rule, any value can be calculated, so the initial terms are irrelevant).

Press Num and compare the sequences.
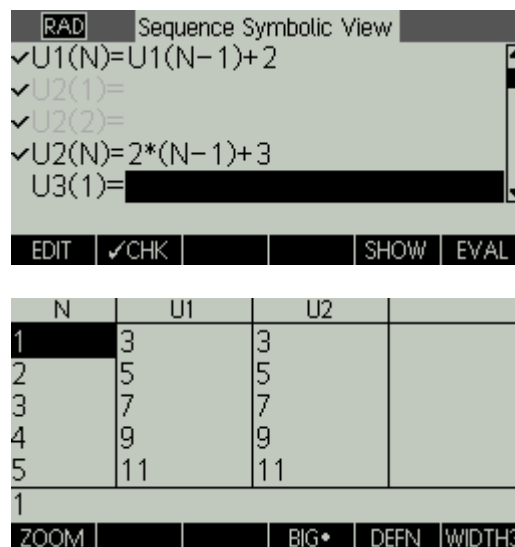
This provides an excellent platform to explore different sequences and practice finding position-to-term rules to match term-to-term rules.

Now that we know the position-to-term rule, we can find values for the corresponding series.

To find 3+5+7+9+11, we are summing terms in the sequence from $n = 1$ to $n = 5$

So, we find $\sum_{1}^{5} 2(n-1) + 3$

Press Math, navigate to Loop and choose $\sum$
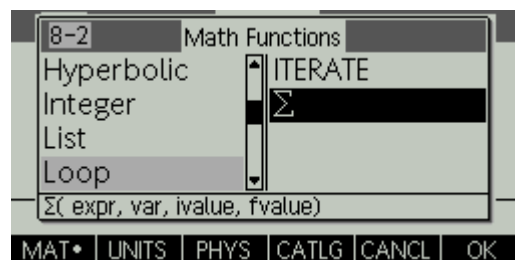
Note that on the menu, it tells you how to enter the values. Click OK, then enter the expression $2(n-1) + 3$ using the ALPHA key to enter the N. The *var* is N and the limits are 1 then 5. Make sure to put the commas in between each entry.
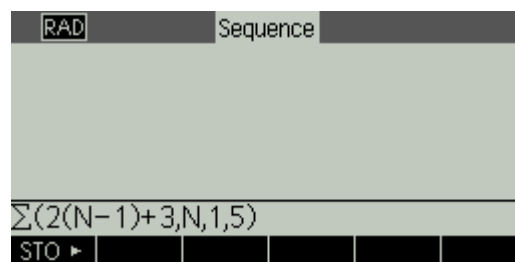
Now press ENTER to evaluate.

(This is how it appears with textbook display switched on).

Check that you agree with the result! (3+5+7+9+11 = 35)
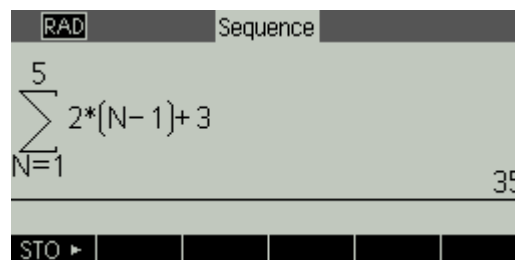
You can easily change the values by using the cursor to select the entry, then pressing F4 for copy and editing. You should explore the effect of making changes to the limits and to the rule.

Exploring Maths at Advanced Level

You can easily explore more complicated sequences. An excellent example would be Fibonacci sequences.

1, 1, 2, 3, 5, 8, 13, …

Here, we know the first and second terms and a rule that each term is the sum of the preceding two.

Relaunch the sequence applet and reset it, ready to enter the new sequence.

We have U1(1)=1, U1(2)=1 and U1(N)=U1(N-1)+U1(N-2)

Check that it has worked by looking at the table.

(Press Num)
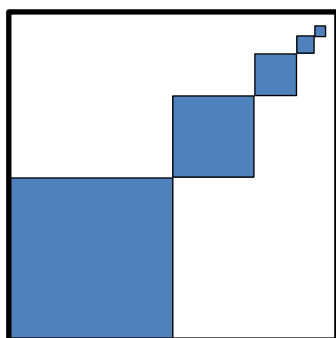
A square is divided into quarters and one quarter is shaded. A quarter of the opposite unshaded quarter is shaded. A quarter of this quarter is shaded and so on as shown. What fraction of the original square is shaded?

If the original square has unit area, then the shaded part is $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

That is, adding up the terms of the sequence with

$$u_n = \left(\frac{1}{4}\right)^n$$

So, see what happens to this value as n increases.

Enter $\sum_1^1 \left(\frac{1}{4}\right)^n$

Press $a \leftrightarrow b/c$ to see the outcome as a fraction.

Use the up arrow to COPY and EDIT the sum. Keep increasing the last term to see how the sum is changing. What value does it tend towards as the number of terms increases?

Try to explain how you know that the shaded area must be this fraction of the whole.

6: Projectiles. Parametric Functions

We can set up graphs with variable coefficients. In this way we can explore the effect of changing the coefficients. A good example of this is to graph a general projectile.

We know the horizontal equation of motion is $x = utcos(A)$

The vertical equation is $y = utsin(A) - 0.5gt^2$

Where u is the initial velocity, A the angle of projection, g acceleration due to gravity and t the time taken.

So, we have two parametric equations with parameter t

First, we enter the equations: (note that T is the parameter so use the X,T,θ,N key).

Press Apps, find Parametric, press RESET then OK, then START

For X1(T) type ALPHA U × T × COS(ALPHA A) then OK

For Y1(T) type ALPHA U × T × SIN(ALPHA A) − 0.5 × ALPHA G × T2 then OK

If you press Plot now, you will see nothing, since the value of all of the variables defaults to zero. So, now we assign values to the variables. First, we will take $g = 9.8$
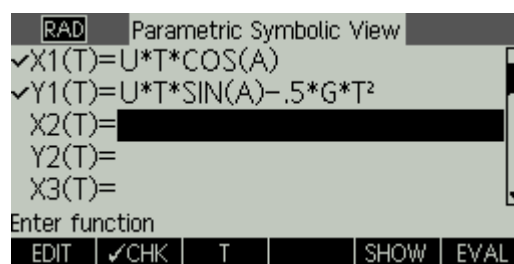
Press HOME, type 9.8 press STO ALPHA G ENTER

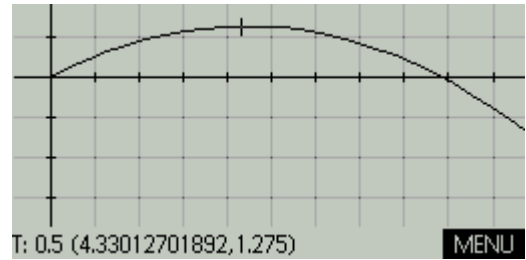Now choose some sensible values for the other variables, (say):

30 STO ALPHA A ENTER

5 STO ALPHA U ENTER

Press SHIFT MODES and set the Angle Measure to degrees.

Now press Plot again.
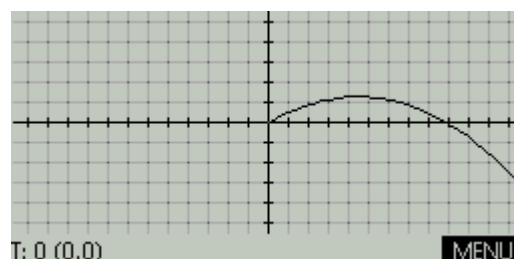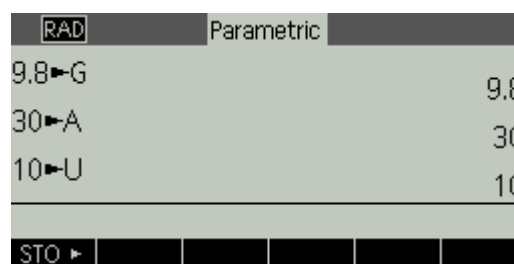
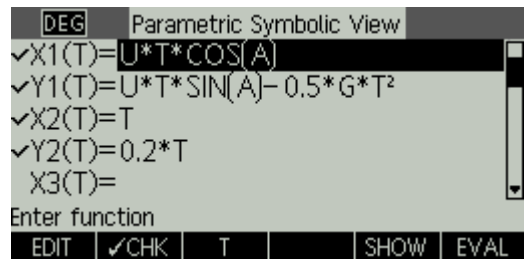We can use ZOOM Box… to zoom in on the motion and use TRACE to find key points.

We can go back to the original zoom level using ZOOM Decimal.

Having set this up you can now explore changing the variables. How much difference does it make if we double the initial angle?
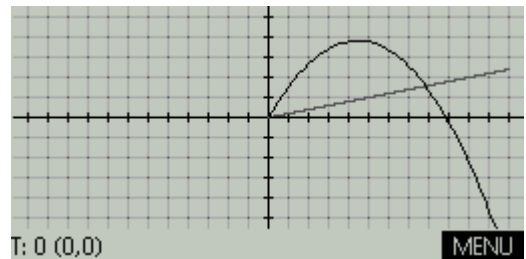
Press Home and 60 STO ALPHA A ENTER, then Plot.
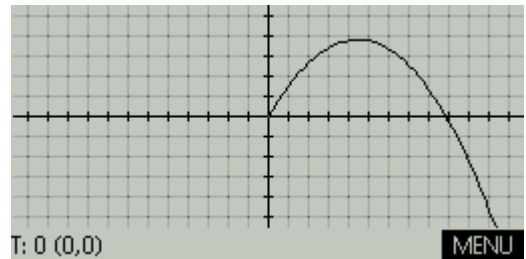
We can solve sloping ground problems. For example, suppose the projectile is launched from a point on a slope with gradient 0.2. We can model this with a function $f(x) = 0.2x$. However, we are in parametric mode, so we use the parameter T for X as $x(t) = t$ and then $y(t) = 0.2t$ so we enter X2(T)=T and Y2(T)=0.2T.
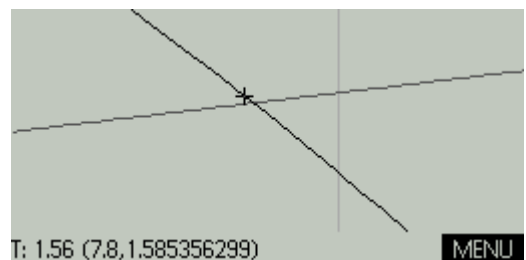
Press Plot to look at the motion.

Now use zoom box to find out when the projectile lands.

First use zoom box to zoom in close on the landing point. Then TRACE to find the value of T. You can increase the precision by changing the T steps using SHIFT Plot. To 2 d.p. the projectile lands after 1.56 seconds.

Finally you can even see what happens going back in time! Press SHIFT plot to Setup the plot. The T range defaults to starting at time T=0, which is logical. But change the TRNG to start in the past e.g. T=-2 …
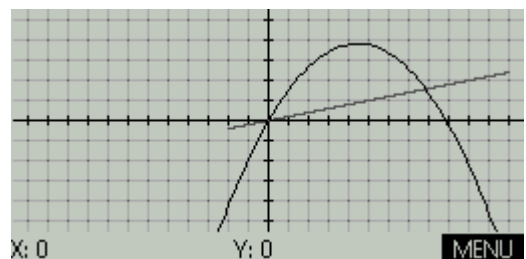
7. Statistics: Probability Distributions and Inference

For statistical work, you should use the three statistics applets:

- 1 variable statistics (for box plots, histograms etc. with central tendency and spread)
- 2 variable statistics (for scatter plots, correlation and regression)
- Inference (for inferential statistics and confidence intervals)

Also, press Math and look at the following sub menus:

- Distribution (binomial, normal, poisson, etc.)
- Probability (permutations, combinations, random numbers, etc.)

One Variable Data

Get started by entering some data for 2 single variable lists.

Apps/Statistics 1Var/RESET/OK/START

For example, the ages of visitors to a swimming pool on Friday are as follows:

{12, 13, 12, 12, 12, 12, 3, 38, 12, 13, 12, 13, 12, 12, 12, 13}
enter this under D1

… and on Saturday are as follows:

{3, 56, 23, 12, 14, 15, 7, 5, 35, 28, 17, 2, 6, 15, 21}
enter this under D2

Press Symb and press CHOOS to change Plot 1 and Plot 2 from Histogram to BoxWhisker (notice the wide range of other plot types … experiment to see what they are like)

Also, enter D2 for the list associated with plot H2.

Press Plot (then MENU/ZOOM/Autoscale) to compare the two distributions.

Press Num and press F6 for STATS to see the summary statistics. Press down to see mean and standard deviation(s).
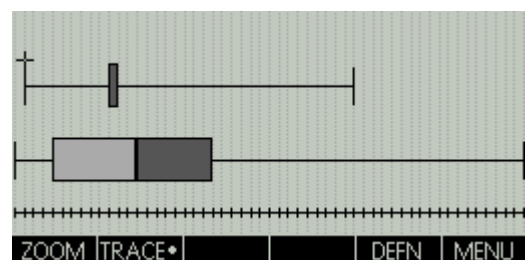
Click OK to exit the STATS view.

X	H1	H2	
n	213	259	
Min	3	2	
Q1	12	15	
Med	12	23	
Q3	13	35	
213			

X	H1	H2	
ΣX	3569	7317	
ΣX²	80967	278725	
x̄	1.67559E1	2.82510E1	
sX	9.991814	1.67069E1	
σX	9.968331	1.66746E1	
3569			

For grouped data use one list e.g. D1 for the values and a second list for the frequencies e.g. D2.

For example, here is a distribution showing the sizes of mens' shoes:

| Size	5	6	7	8	9	10	11	12	13
Frequency	7	13	21	35	39	28	21	8	1
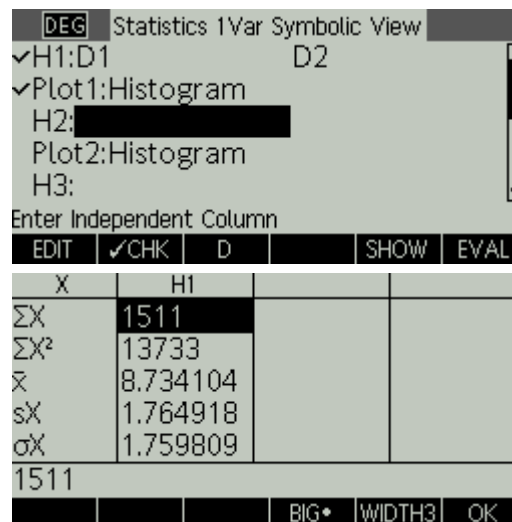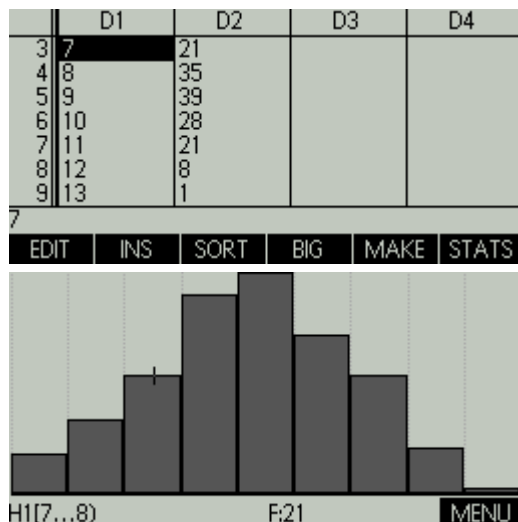
Two Variable Data

Start the 2 variable statistics applet: Apps/Statistics 2Var/RESET/OK/START

Enter some paired data, e.g. data comparing shoe size with handspan:

Shoe Size	6	9	11	9	10	12	9	7	5	12	9	7	8	9	4	8
Hand Span (cm)	18	23	25	22	26	27	24	23	19	28	21	22	20	24	19	20

Enter Shoe size as C1 and Handspan as C2

Press Symb and notice that the defualt is to plot C1 against C2 which is what we want.
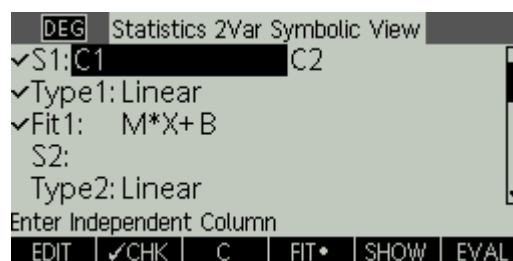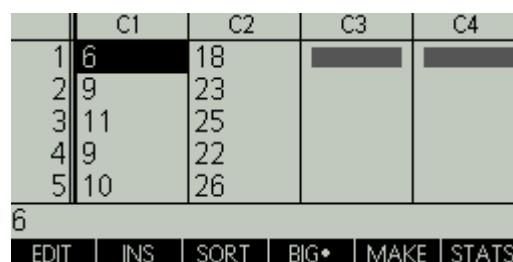
Press Plot/MENU/ZOOM/Autoscale to see a scatter plot with the linear regression line. Clicking FIT will turn the line on and off.
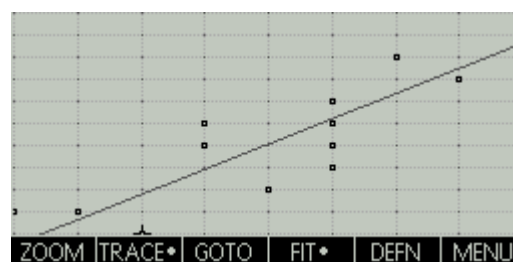
Press Num and STATS to see the summary statistics with the correlation coefficient and covarience.

Notably, the correlation coefficient $r = 0.87$ (t 2 d.p.) shows a reasonably strong relationship.

Press Symb again and notice that the details for the regression line are now shown.
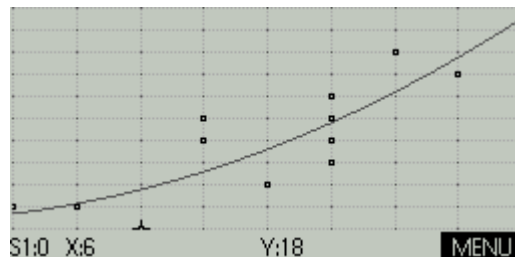
The line is $f(x) = 1.14x + 12.9$ (to 3 s.f.) and our x values (i.e. the first list) showed shoe size. This suggests a model of the relationship between show size and handspan as:

$H = 1.14S + 12.9$ where $H =$ handpsan and $S =$ shoesize.

If we want to see if we would get a better fit with a different model, then we can change the model with the Type options. Navigate to the Type1 entry and click CHOOS. Press Symb to see the new model. Press Num and STATS to calculate the statistics (many of which are undefined in this new model) and press Symb again to see the coefficients for the new model. Click EVAL to improve the view …

This gives us a new model of $H = 0.94S^2 - 0.4S + 18.8$ where $H =$ handpsan and $S =$ shoesize.

Confidence Intervals

We have some data about the shoe sizes of thirty female maths students. We want to use this data to answer the question, "What is the mean shoe size for UK female maths students?"

Here we are using a sample to 'infer' something about the whole population. This is called inferential statistics. The issue will be how confident we can be in the outcomes. We can say that with a given level of probability, the mean will lie within a certain interval. At a 95% level of confidence, we can calculate the range within which the mean will lie, with a probability of 0.95.

To do this, we need to know the mean and standard deviation and the number of data values of the sample.

Exploring Maths at Advanced Level

First, we enter the sample data into the 1 variable statistics applet.

The data is: {2, 7, 6, 4, 5, 3, 4, 5, 6, 7, 4, 3, 5, 4, 4, 3, 5, 6, 6, 8, 6, 7, 8, 4, 3, 9, 5, 7, 6, 4}

Press Apps, choose Statistics 1Var then RESET/OK/START

Enter the data into list D1

Now start the inference applet:

Press Apps, choose Inference then RESET/OK/START

Click CHOOS to change the Method to Confidence Interval and the Type to T-Int 1μ

Press Num to get the statistics we entered earlier.

Click F4 for IMPRT. Check that we will import list D1 from the Statistics 1Var applet (which it will be expecting) and click OK

You can now see the sample mean $\overline{x}$, the sample standard deviation s and the number of sample data points n. You need to change the confidence interval C to 0.95

Press F6 for CALC to make the calculations.

This tells us that the probability is 0.95 that the shoe sizes of the whole population will fall between 4.55 and 5.85 (to 2 d.p.)

Press Plot to see this graphically.

As usual, experiment! See what impact changing the confidence interval has. Look at the other tests available. Research them. See what difference they make.

8. Calculus

It is important to remember that the HP39gII is a numeric calculator. If it could differentiate and integrate for you symbolically, then you wouldn't be allowed to use it in your exams. What it will let you do is to explore how calculus works.
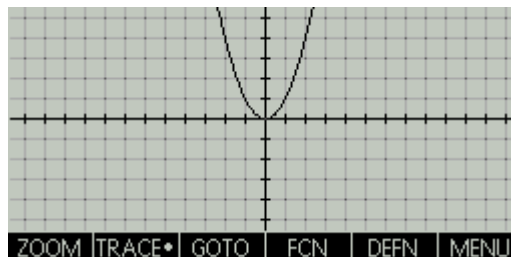
Differentiation

Let's explore the differential of $f(x) = x^2$. The differential is the function which gives values of the slope of the original function at all values of x.
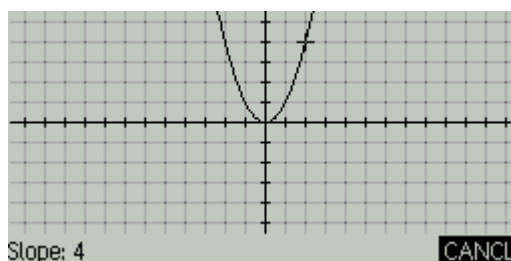
Open and reset the Function applet:

Apps/Function/RESET/OK/START

Enter the function $f(x) = x^2$

Press plot to see the graph.

Make sure that TRACE is on (the little blob is showing next to TRACE) and move the cursor to the furthest right hand/positive end of the visible graph. Now click F4 for FCN and choose Slope.

Before you move the cursor, guess what the slope values will be as you move the cursor to the left hand/negative end of the visible graph. When you have formed a good idea, move the cursor and see.

Look to see the relationship between the value of the slope and the value of x. (It is easiest to see at key points like the one shown where the slope is 4 and you can see the value of x is 2).
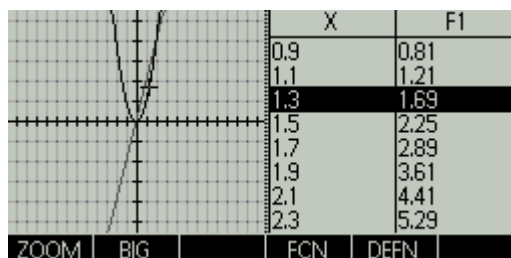
You can see the relationship more clearly by splitting the screen. Press Views. Choose the second option (Split Screen: Plot table). Click F4 for FCN and choose slope.
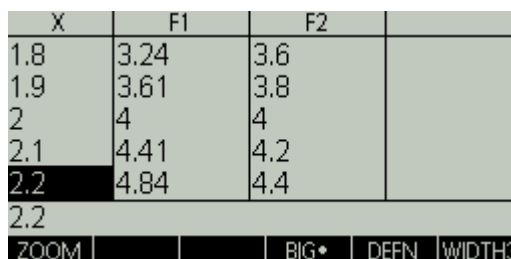
Jot down the points you have found and decide what function shows the relationship between x and the slope. Go back to the Symb page and enter this function for F2(X). Press Num and check that the values for F2(X) are the same as the ones you jotted down.

Repeat this process with a range of functions. The sine function is very interesting. Make slight changes to the functions and see what difference it makes.

(e.g. change $f(x) = x^2$ to $f(x) = x^2 + 1$)

Get a good feel for the differential functions generated by different types of functions.
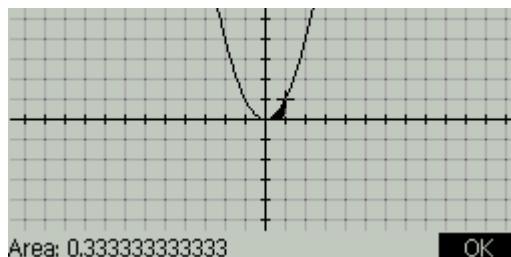
Integration

You can apply the same process to get a feel for the integral function. This time we are looking at the area under the graph.
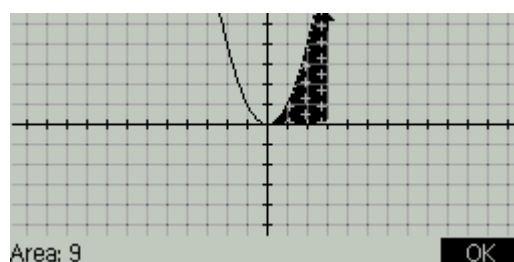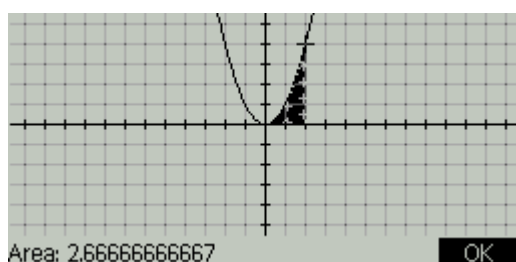
Launch and reset the function applet.

Enter the function $f(x) = x^2$

Press MENU then FCN and select signed area. The cursor will already be at the origin, if not, move it there. Click OK. Now move the cursor to $x = 1$ and click OK.

Area: 0.333333333333   OK

Now repeat the process starting from $x = 0$ to $x = 2$ … then from $x = 0$ to $x = 3$

Area: 2.66666666667   OK   Area: 9   OK

Writing the values as fractions makes things clearer:

x	0	1	2	3
Area from 0 to x	0	$\dfrac{1}{3}$	$\dfrac{8}{3}$	$\dfrac{27}{3}$

Decide what you think the area function (the integral) is, Press Symb and enter it as F2(X). Press Num and check that this generates the same values.

X	F1	F2	
1.8	3.24	1.944	
1.9	3.61	2.286333	
2	4	2.666667	
2.1	4.41	3.087	
2.2	4.84	3.549333	
1.8			

ZOOM   BIG•   DEFN   WIDTH3

Press Plot and TRACE over F2(X) to check.

X: 2   F2(X): 2.66666666666   MENU
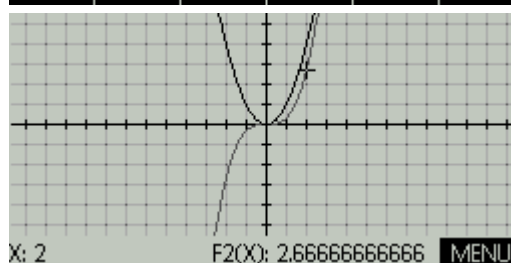
Again, do this with different types of functions. Make small changes to existing functions. The aim is to get a feeling for how the area function varies. To be able to work out exactly what the function is from this small amount of data would be very difficult for most functions. But the key point is to experiment and get that all important feeling for how calculus works.

Notes

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