

# Pythagoras: Solving Triangles

## What is the Overall Purpose?

- Finding the lengths of sides and the sizes of angles in triangles.

## How Does that break down?

- Finding sides in a right angled triangle: Pythagoras Theorem
- Finding sides and angles in right angles angled triangles where the unknowns are sides or angles.
- Finding sides and angles in non-right angles angled triangles

## What are trigonometric functions?

- We define trigonometric functions as the ratio of sides in a right angled triangle
- Trigonometry
- Periodic functions

## Sources of application

- Surveying and mapping (the principle of triangulation using a theodolite)
- Astronomy and it's application to navigation
- <http://www.hps.cam.ac.uk/starry/mathematics.html>

## Lesson Planning

For this sequence of lessons the worksheets have been laid out to contain an implicit plan.

- *Learning Objectives*: contained in the scheme below.
- *Starter Activity*: some of the worksheets have two activities, therefore the first is intended as a starter. In some cases e.g. the Pythagoras investigation, the main activity will take the whole lesson. Otherwise a short set of quick questions re-capping the ideas of the previous lesson should be used.
- *Main Activity*: each sheet contains a main activity/exercise.
- *Exposition*: the summary and context material on the sheet is intended for exposition.

## Module Scheme of Work

Lesson	Title	Learning Objectives
1	Pythagoras: investigation	<ul style="list-style-type: none"><li>• To recognise square numbers.</li><li>• To identify right angled triangles</li><li>• To develop relationships between the squares of sides a triangle and the size of its angles.</li></ul>
2	Pythagoras theorem calculations	<ul style="list-style-type: none"><li>• To be able to calculate the third side of a right angled triangle when the other two are known.</li><li>• To recognise Pythagorean triples</li></ul>
3	Problem Solving	<ul style="list-style-type: none"><li>• To be able to recognise problems that can be solved using Pythagoras' theorem.</li><li>• To be able to solve such problems.</li></ul>
4	Clinometer	<ul style="list-style-type: none"><li>• To use a clinometer to measure angles of elevation</li></ul>

5	Trigonometric Functions	<ul style="list-style-type: none"> <li>• To recognise that the ratio of sides of a right angled triangle depends only on the angle.</li> <li>• To remember the definitions of the trigonometric functions.</li> <li>• To use a calculator to find the values of trigonometric functions.</li> </ul>
6	Calculating with Sine	<ul style="list-style-type: none"> <li>• To calculate the lengths of sides and angles in a right angled triangle using the sine function.</li> </ul>
7	Calculating with Sine, Cosine and Tangent	<ul style="list-style-type: none"> <li>• To calculate the lengths of sides and angles in a right angled triangle using the sine, cosine and tangent functions.</li> </ul>
8	Graphs of trigonometric functions	<ul style="list-style-type: none"> <li>• To recognise the shape of trigonometric graphs.</li> <li>• To plot and draw trigonometric graphs.</li> </ul>
9	Periodic Functions	<ul style="list-style-type: none"> <li>• To recognise the properties of trigonometric graphs.</li> <li>• To find graphical solutions to trigonometric equations</li> </ul>
10	Transposition of graphs	<ul style="list-style-type: none"> <li>• To recognise the effect of transpositions of trigonometric graphs.</li> </ul>
11	The cosine rule	<ul style="list-style-type: none"> <li>• To calculate the length of unknown sides in non-right angled triangles using the cosine rule.</li> </ul>
12	The sine rule	<ul style="list-style-type: none"> <li>• To calculate the length of unknown sides and angles in non-right angled triangles using the sine rule.</li> </ul>

## Worksheet A1: Pythagoras Investigation

You will need:

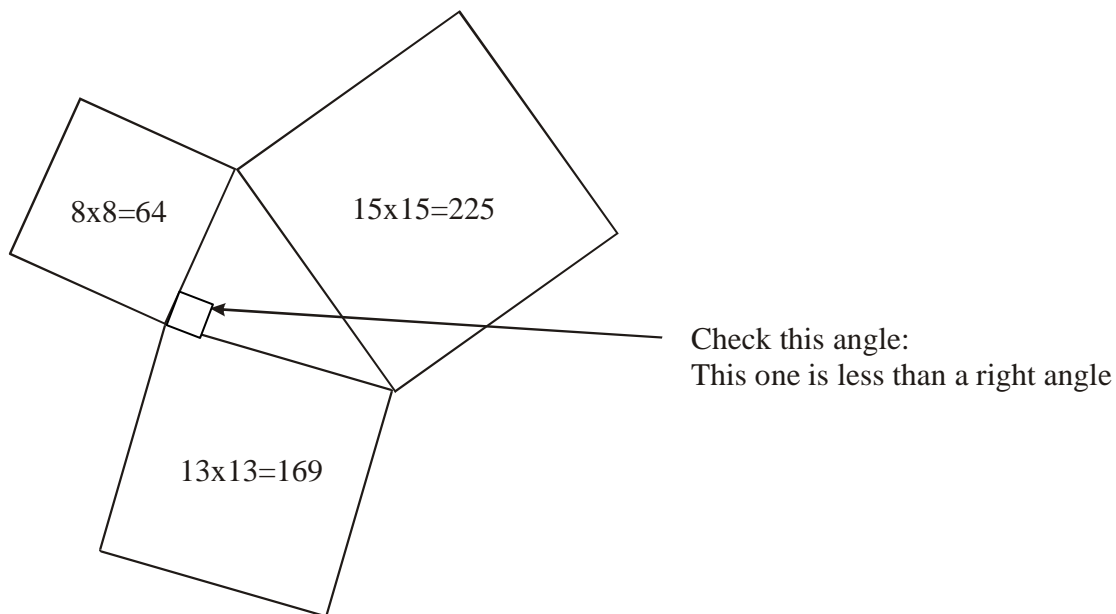
- Sheets of squared paper, scissors, a set square.

You need to make:

- 20 paper squares. One each of  $1\text{cm}\times 1\text{cm}$ ,  $2\text{cm}\times 2\text{cm}$ ,  $3\text{cm}\times 3\text{cm}$  up to  $20\text{cm}\times 20\text{cm}$ .

You need choose sets of 3 squares to fit together. Fit corner to corner to leave a triangular space inside.

Your aim is to find sets of three squares, which make a right angle triangle inside (There are only 5 to find). Use the set square to test if the largest angle is  $90^\circ$  or not. **BE VERY ACCURATE!**



Copy and complete the table:

Smallest Square	Middle Square	Largest Square	Equal, greater or less than $90^\circ$
$8\times 8=64$	$13\times 13=169$	$15\times 15=225$	Less

Conclusions:

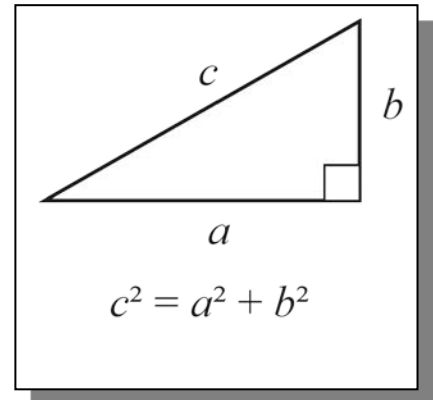
- Look at the squares which give a right angle. Find a rule connecting them.
- Find a rule to work out if the angle is equal, greater or less than  $90^\circ$ .

## Worksheet A2: Pythagoras Theorem Calculations

When the angle is  $90^\circ$

If you add up the area of the two smaller squares you get the same as the area of the largest square.

This is called Pythagoras' Theorem after the Greek Mystic, Numerologist and Mathematician, Pythagoras of Samos. The theorem was known long before the time of Pythagoras. It appears in ancient Egyptian writing. There is evidence that ancient Egyptian farmers used the rule to make sure that their fields were at  $90^\circ$  to the river Nile.



This is one you could have found:

<i>Length</i>	6		8		10
Square	<input type="text" value="36"/>	+	<input type="text" value="64"/>	=	<input type="text" value="100"/>

The numbers 6, 8 and 10 fit Pythagoras theorem.  
6, 8, 10 is called a **Pythagorean Triple**

If you know the square and you want to find the length. You can use the square root button  $\sqrt{\quad}$  on your calculator.

## Exercise

Copy and fill in: (*Hint*: work out the missing squares first)

1.

<i>Length</i>	9		12		
Square	<input type="text"/>	+	<input type="text" value="144"/>	=	<input type="text" value="225"/>

2.

<i>Length</i>	3		4		
Square	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

3.

<i>Length</i>	5				13
Square	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

4.

<i>Length</i>	7		24		
Square	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

5.

<i>Length</i>			20		25
Square	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

6.

<i>Length</i>	5		7		
Square	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

Use the  $\sqrt{\quad}$   
(Square root)  
button)

7.

<i>Length</i>	4				10
Square	<input type="text"/>	+	<input type="text"/>	=	<input type="text"/>

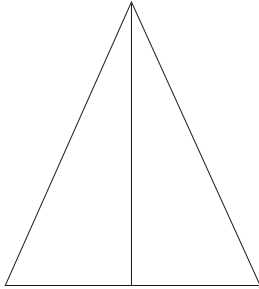
8. Write a list of any other Pythagorean Triples you found.

## Worksheet A3: Problem Solving

Pythagoras' theorem allows us to work out the third side in any right-angled triangle if we know the lengths of the other two.

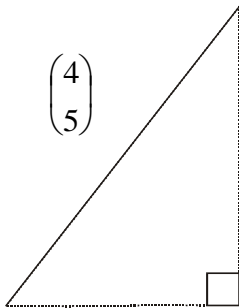
If there is a problem to solve which includes lengths in a right-angled triangle it is quite likely that Pythagoras will be useful.

Look out for them!

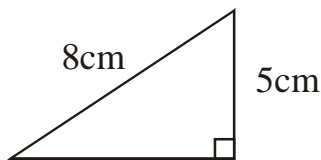


An isosceles triangle can be turned into two right angled triangles.

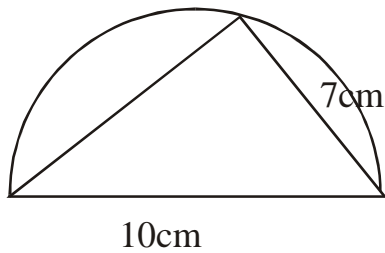
Then you can use Pythagoras' theorem.



A vector gives a right-angled triangle. You can find the length of a vector with Pythagoras theorem.



To find the area of this right-angled triangle, you will need to work out the third side first. Use Pythagoras' theorem.

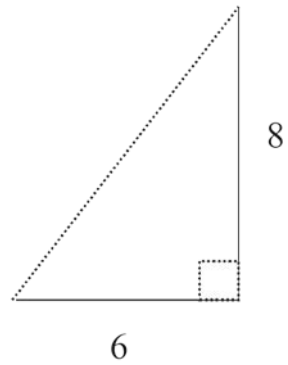


Look for hidden right angles. In a semi circle the angle at the circumference is always a right angle.

So, we can use Pythagoras' theorem to find the length of the third side.

## Exercise

1. Calculate the length of the vector  $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$



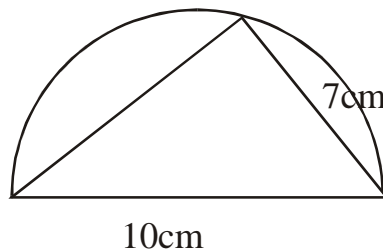
2. Calculate the length of the vectors:

(a)  $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$

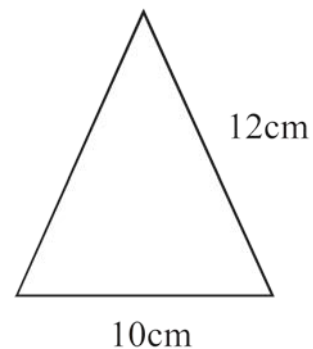
(b)  $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

(c)  $\begin{pmatrix} -3 \\ 8 \end{pmatrix}$

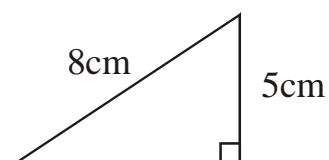
3. Calculate the length of the missing side in this semi-circle:



4. Calculate the height of this isosceles triangle.  
(Hint: divide the length of the base by 2)

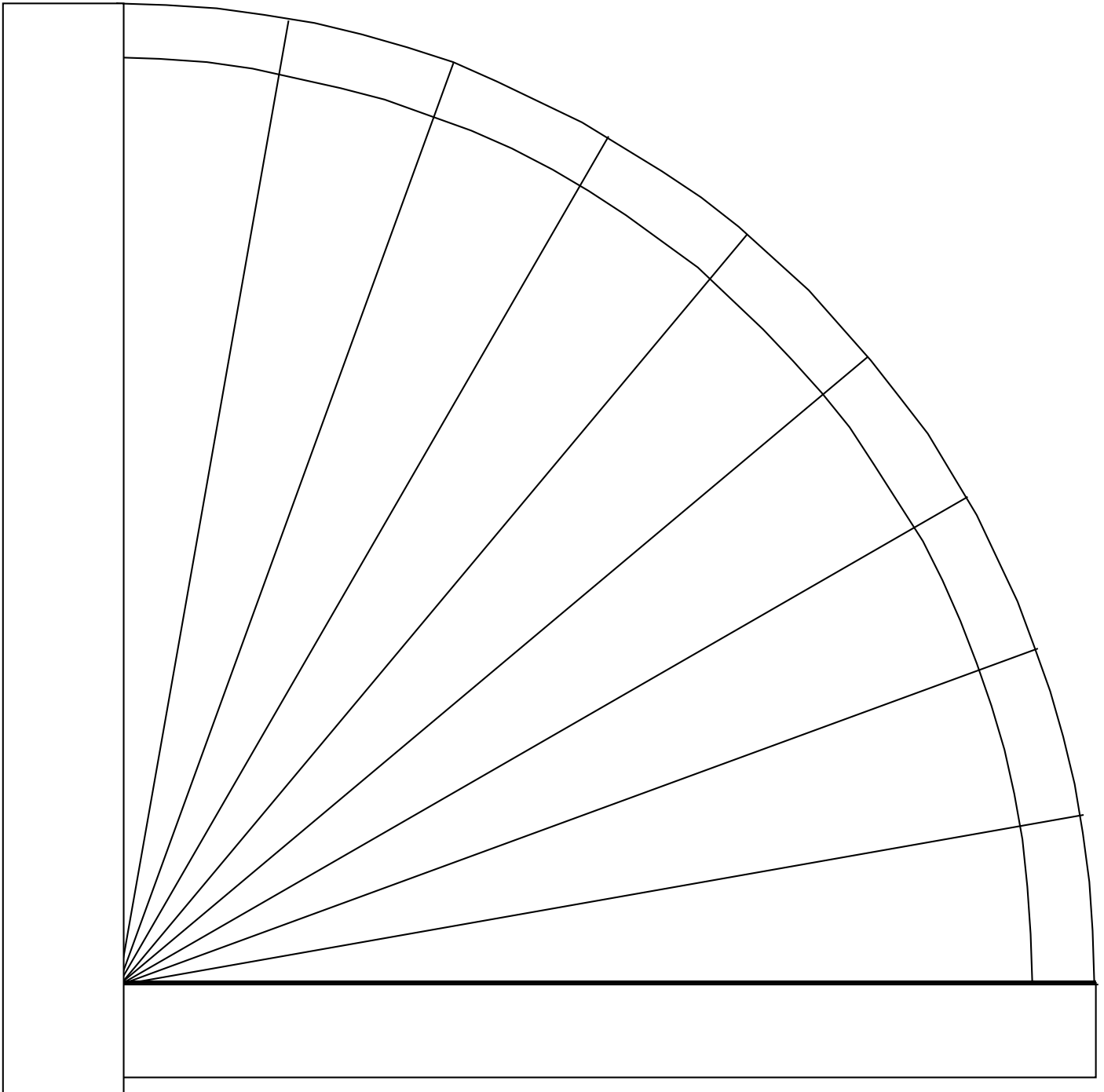


5. Calculate the area of this right angled triangle:



## Worksheet A4: Clinometer

**You will need:** scissors, tape/glue, pins, this page printed onto card, an extra sheet of card.

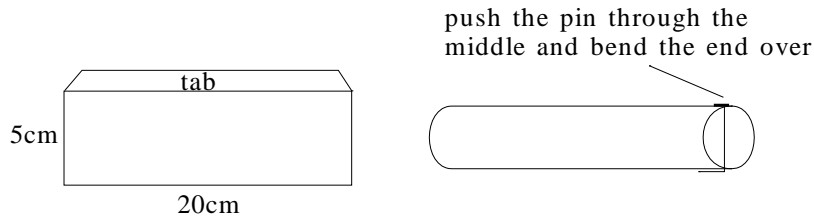




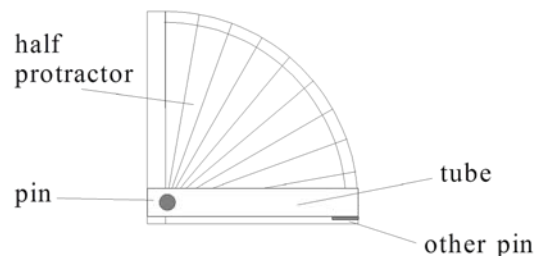
Work in a small group.

Each group should first make a clinometer:

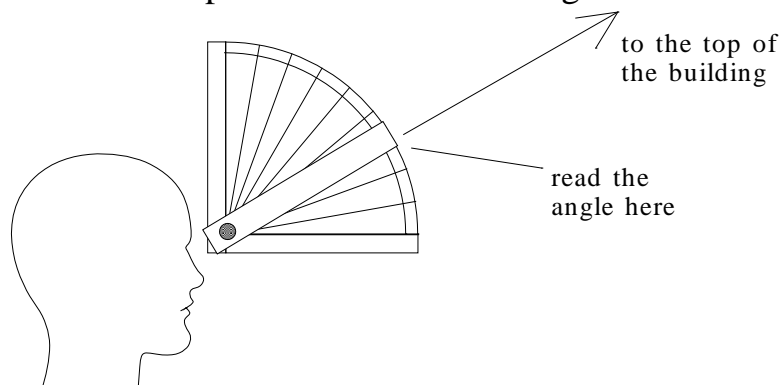
1. Cut out the quarter circle protractor. Mark on the angles (from  $0^\circ$  to  $90^\circ$ ).
2. Cut out a card rectangle roughly  $20\text{cm} \times 5\text{cm}$  with a tab along one of the longer sides. Roll it into a long thin tube and glue down the tab. Push a pin through the middle of the tube near to the end.



3. Fit the tube to the quarter circle by pushing a pin through the other end of the tube, through the centre of the circle. Bend the end of the pin over and tape it down.

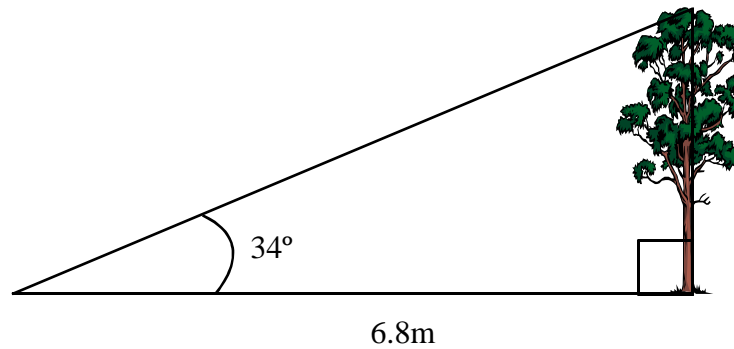


4. Using your clinometer:
  - (a) Hold the clinometer with the tube at your eye.
  - (b) Hold the quarter circle exactly level. ( $0^\circ$  must be horizontal)
  - (c) Look into the tube.
  - (d) Turn the tube until you can see the top of the building (the two pins should form a cross in the centre of the tube).
  - (e) Hold the tube in position and read the angle.



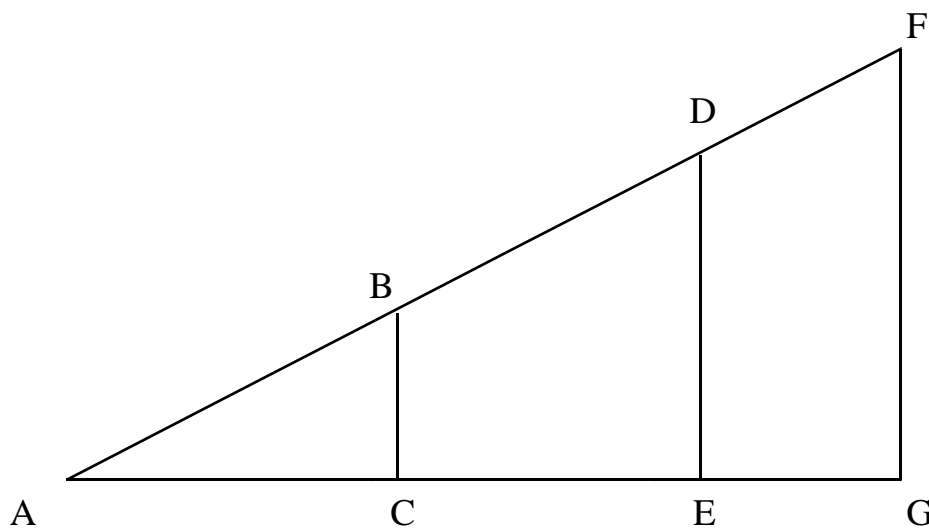
5. Take your clinometer to an open place where you can see building, trees etc. close by. Measure your distance from the bottom of the building, tree etc. Measure the angle to the top with your clinometer. Make a table of results.

## Worksheet A5: Trigonometric Functions



You have collected information to draw diagrams like this. Notice that the triangle we can draw is a right angle triangle. If we could calculate the height of the triangle, then we would know the height of the tree. To do this we can use **trigonometry**. Trigonometry is the study of the measurement of angles. We will look at 3 trigonometric functions: Sine, cosine and tangent. These tell us the result when you divide different pairs of sides in a right angled triangle.

### Activity



The diagram shows three triangles: ABC, ADE, AFG. Very accurately measure the lengths of all of the sides. Copy and complete the table on the next page: (Write your divisions to 2 decimal places).

	AB	BC	AC	$BC \div AB$	$AC \div AB$	$BC \div AC$
--	----	----	----	--------------	--------------	--------------

ABC						
	AD	DE	AE	$DE \div AD$	$AE \div AD$	$DE \div AE$
ADE						
	AF	FG	AG	$FG \div AF$	$AG \div AF$	$FG \div AG$
AFG						

Look at the three columns with divisions. What do you notice?

- The first column showed the side opposite the angle divided by the hypotenuse. We call this the sine of the angle. (Abbreviation: **sin**)
- The second column showed the side adjacent to the angle divided by the hypotenuse. We call this the cosine of the angle. (Abbreviation: **cos**)
- The third column showed the side opposite the angle divided by the side adjacent to the angle. We call this the tangent of the angle. (Abbreviation: **tan**)

To find the sine of an angle (e.g.  $47^\circ$ ) on a calculator we press:  $\sin 47 =$

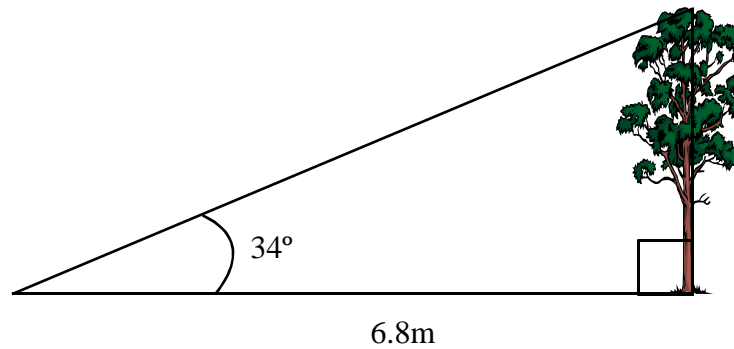
To find the angle which has a given sin (e.g. 0.46) we press:  $2^{\text{nd}} \sin^{-1} 0.46 =$

### Exercise

Use your calculator to find:

- (a)  $\sin 10^\circ$                       (b)  $\sin 73^\circ$                       (b)  $\sin 18^\circ$
- (a)  $\sin 0^\circ$                             (b)  $\sin 90^\circ$                       (b)  $\sin 270^\circ$
- (a)  $\sin 100^\circ$                         (b)  $\sin 30^\circ$                       (b)  $\sin -30^\circ$
- (a)  $\sin^{-1} 0.5$                         (b)  $\sin^{-1} 1$                         (b)  $\sin^{-1} 0.72$
- Find an angle which has  $\sin 0.671$  ?

## Worksheet A5: Trigonometric Functions



You have collected information to draw diagrams like this. Notice that the triangle we can draw is a right angle triangle. If we could calculate the height of the triangle, then we would know the height of the tree.

To do this we can use **trigonometry**.

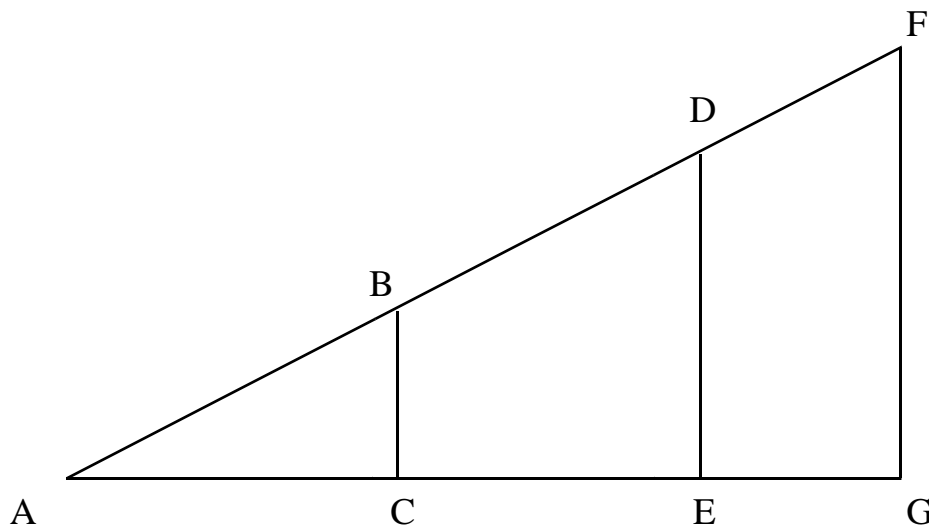
Trigonometry is the study of the measurement of angles.

We will look at 3 trigonometric functions:

Sine, cosine and tangent.

These tell us the result when you divide different pairs of sides in a right angled triangle.

### Activity



The diagram shows three triangles: ABC, ADE, AFG. Very accurately measure the lengths of all of the sides. Copy and complete the table on the next page: (Write your divisions to 2 decimal places).

	AB	BC	AC	$BC \div AB$	$AC \div AB$	$BC \div AC$
ABC						
	AD	DE	AE	$DE \div AD$	$AE \div AD$	$DE \div AE$
ADE						
	AF	FG	AG	$FG \div AF$	$AG \div AF$	$FG \div AG$
AFG						

Look at the three columns with divisions. What do you notice?

- The first column showed the side opposite the angle divided by the hypotenuse. We call this the sine of the angle. (Abbreviation: **sin**)
- The second column showed the side adjacent to the angle divided by the hypotenuse. We call this the cosine of the angle. (Abbreviation: **cos**)
- The third column showed the side opposite the angle divided by the side adjacent to the angle. We call this the tangent of the angle. (Abbreviation: **tan**)

To find the sine of an angle (e.g.  $47^\circ$ ) on a calculator we press:  $\sin 47 =$

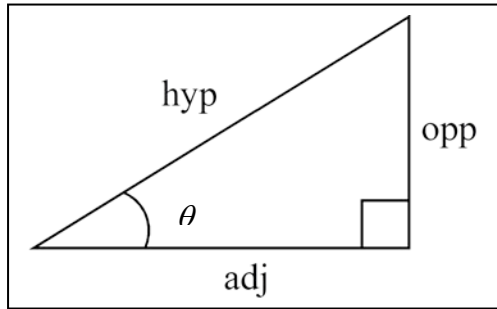
To find the angle which has a given sin (e.g. 0.46) we press:  $2^{\text{nd}} \sin^{-1} 0.46 =$

## Exercise

Use your calculator to find:

- $\sin 10^\circ$
  - $\sin 73^\circ$
  - $\sin 18^\circ$
- $\sin 0^\circ$
  - $\sin 90^\circ$
  - $\sin 270^\circ$
- $\sin 100^\circ$
  - $\sin 30^\circ$
  - $\sin -30^\circ$
- $\sin^{-1} 0.5$
  - $\sin^{-1} 1$
  - $\sin^{-1} 0.72$
- Find an angle which has  $\sin 0.671$  ?

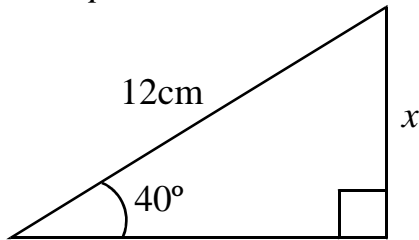
## Worksheet A6: Calculating with Sine



There are 3 different calculations we can make. We can find the missing opposite side, hypotenuse or angle, if we know the other two.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

### Example 1



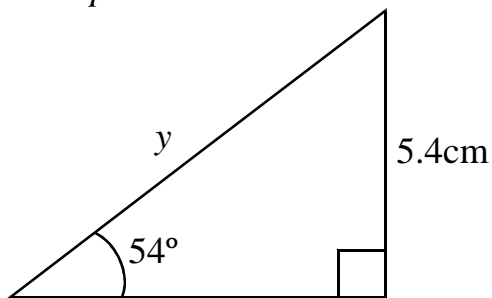
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 40 = \frac{x}{12}$$

$$12 \times \sin 40 = x$$

$$x = 7.7\text{cm (to 1d.p.)}$$

### Example 2



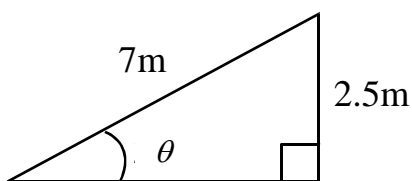
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 54 = \frac{5.4}{y}$$

$$y = \frac{5.4}{\sin 54}$$

$$y = 6.7\text{cm (to 1d.p.)}$$

### Example 3



$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{2.5}{7}$$

$$\sin \theta = 0.357$$

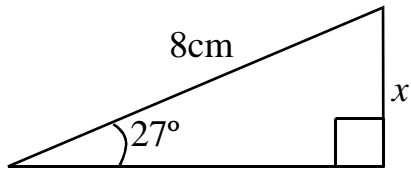
$$\theta = 20.9^\circ \text{ (to 1d.p.)}$$

Use the  $\sin^{-1}$  button on your calculator

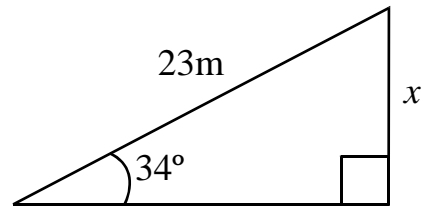
## Exercise

1. Find the size of the side  $x$  to 1 decimal place.

(a)

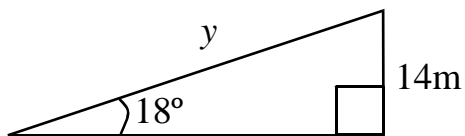


(b)

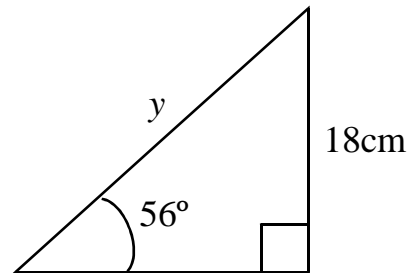


2. Find the size of the side  $x$  to 1 decimal place.

(a)

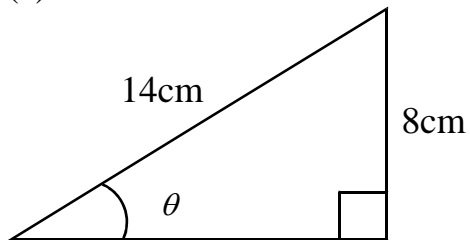


(b)

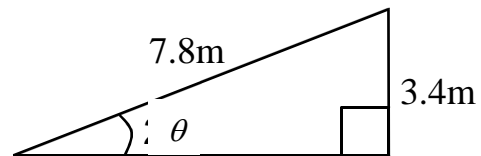


3. Find the size of the angle  $\theta$  to 1 decimal place.

(a)

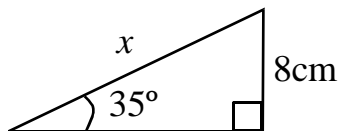


(b)

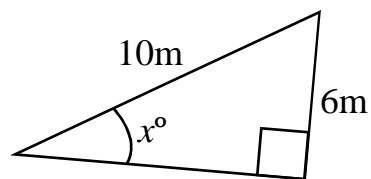


4. Find  $x$  in each case to 1 decimal place.

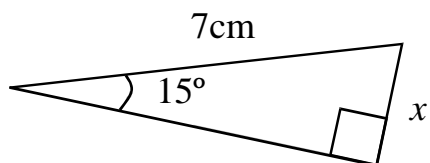
(a)



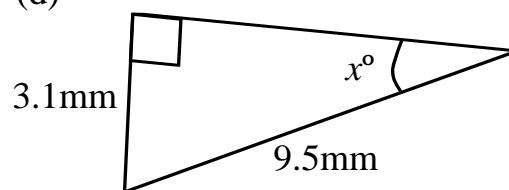
(b)



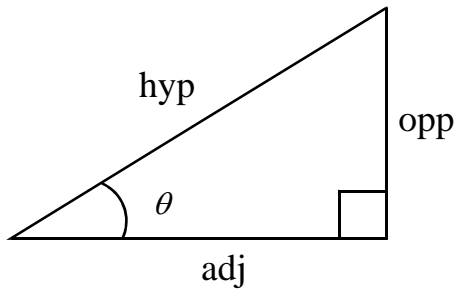
(c)



(d)



## Worksheet A7: Calculating with Sine, Cosine and Tangent



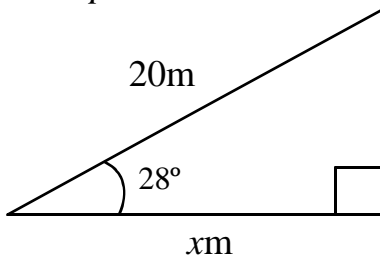
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

Before making a calculation, we must decide which sides are involved. Also, it is very useful to remember all of the ratios. Think of a little rhyme to remember the first letters: SOHCAHTOA.

### Example 1



The two sides involved are **adjacent** and **hypotenuse**.

So we will use **cosine**.

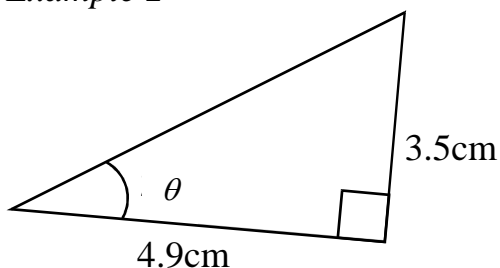
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sin 28 = \frac{x}{20}$$

$$20 \times \sin 28 = x$$

$$x = 9.4\text{m (to 1d.p.)}$$

### Example 2



The two sides involved are **opposite** and **adjacent**.

So we will use **tangent**.

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{3.5}{4.9}$$

$$\tan \theta = 0.714$$

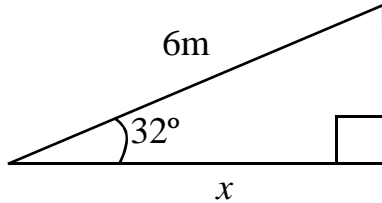
$$\theta = 35.5^\circ \text{ (to 1d.p.)}$$



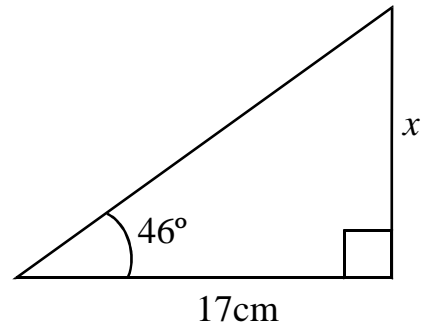
## Exercise

1. Find the size of the side  $x$  to 1 decimal place.

(a)

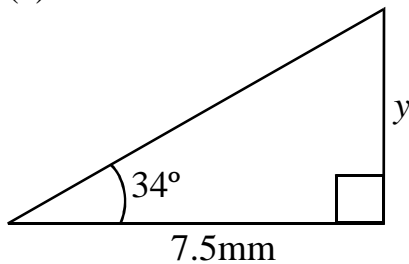


(b)

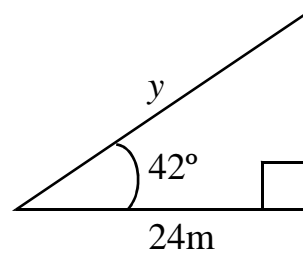


2. Find the size of the side  $x$  to 1 decimal place.

(a)

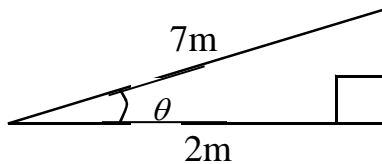


(b)

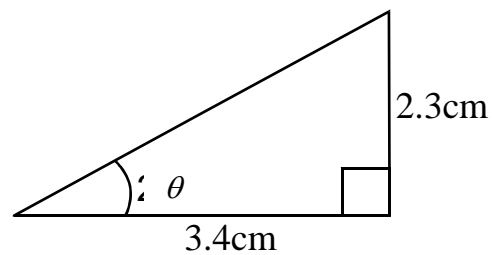


3. Find the size of the angle  $\theta$  to 1 decimal place.

(a)

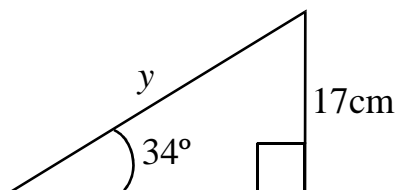


(b)

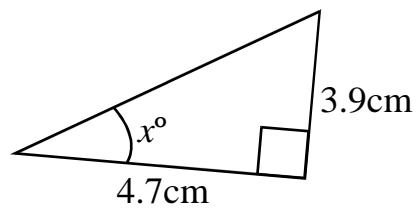


4. Find  $x$  in each case to 1 decimal place.

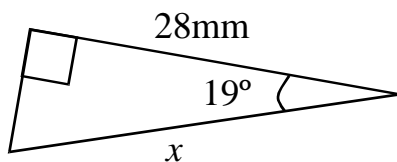
(a)



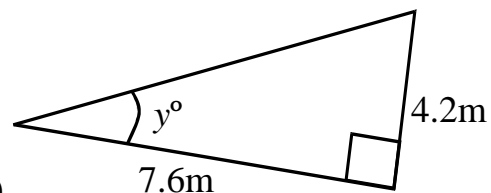
(b)



(c)



(d)



## Worksheet A8: Graphs of Trigonometric Functions

### Activity 1

Use a graphical calculator or graphing software to investigate the graphs of trigonometric functions.

In a graphical calculator press  $Y=$  type  $\sin x$  press GRAPH press ZOOM Trig

- What are the greatest and least values of **sine** and **cosine**.
- What happens to the graph of **tangent** at  $90^\circ$  and  $270^\circ$  ?
- What symmetries to the graphs have ?
- After how many degrees do the graphs repeat themselves?
- Now make variations to the basic graphs e.g.  $\sin 2x$  or  $\sin (x+30)$  or  $2\sin x$  and answer the same questions again.

### Activity 2

**You will need:** graph paper, calculator

Draw graphs of the 3 trigonometric functions:

Use a calculator to fill in the table. Write the values to 1 decimal place.

	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$180^\circ$	$210^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$330^\circ$	$360^\circ$
sin													
cos													
tan													

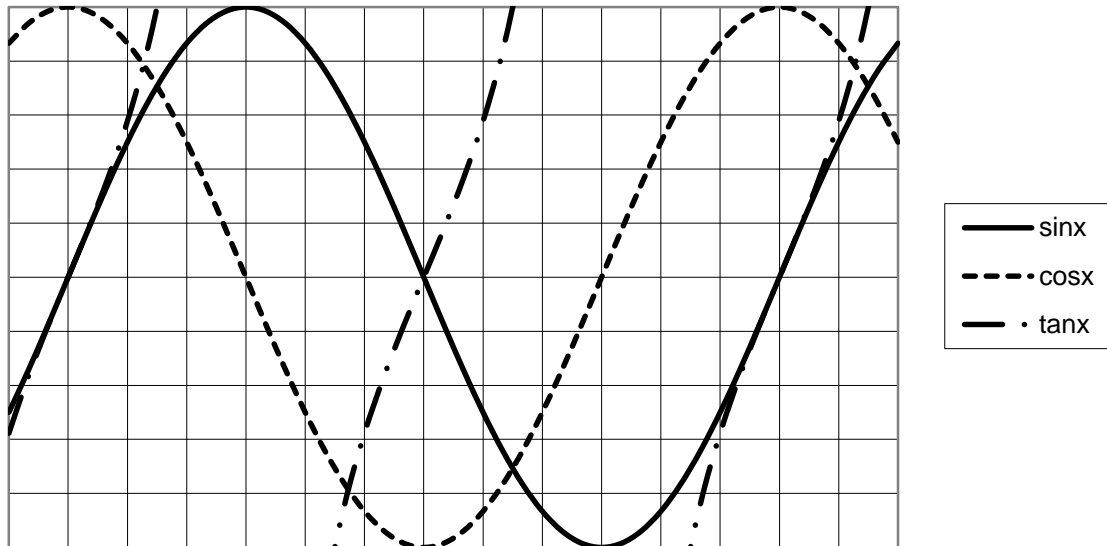
When you have plotted the points for each graph, carefully join them up with a smooth curve.

On the graph of  $\tan$ , draw a dotted vertical line at  $90^\circ$  and at  $270^\circ$ . These lines are called **asymptotes**. They show that the value of  $\tan$  is getting closer to infinity as the angle gets closer to  $90^\circ$  or  $270^\circ$ .

Use your graphs to look up:

1. (a)  $\sin 40^\circ$                       (b)  $\sin 140^\circ$                       (c)  $\sin 320^\circ$
2. (a)  $\cos 25^\circ$                       (b)  $\cos 155^\circ$                       (c)  $\cos 295^\circ$
3. (a)  $\sin 50^\circ$                       (b)  $\cos 40^\circ$                       (c)  $\sin 80^\circ$                       (d)  $\cos 350^\circ$
4. (a)  $\tan 10^\circ$                       (b)  $\tan 170^\circ$                       (c)  $\tan 190^\circ$
5. Write down any observations you have made.

## Worksheet A9: Periodic Functions



Notice that:

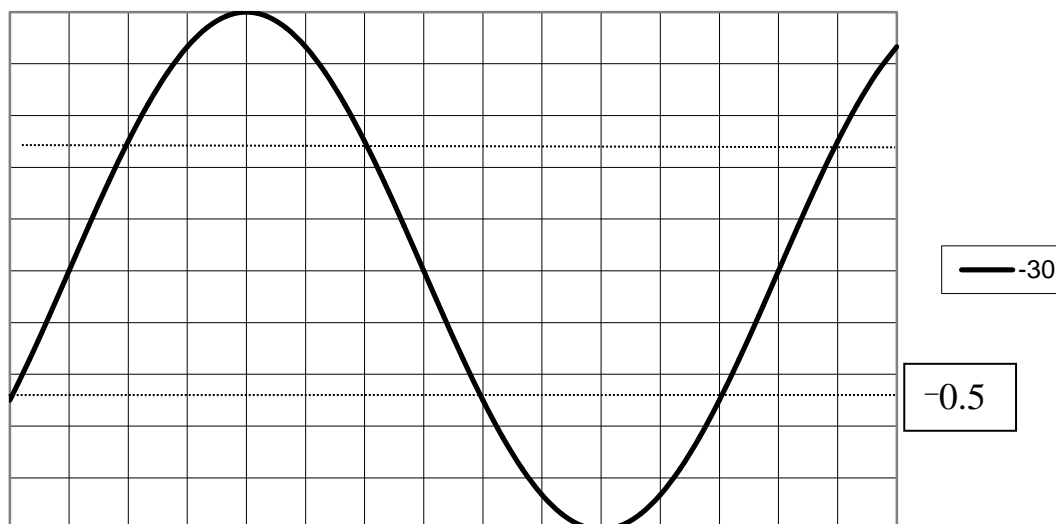
- the value of  $\sin$  and  $\cos$  are always between  $-1$  and  $1$ .
- $\sin$  and  $\cos$  repeat themselves after  $360^\circ$
- $\tan$  repeats after  $180^\circ$
- $\sin$  has  $180^\circ$  rotation symmetry
- $\cos$  has line (mirror) symmetry
- $\tan$  has line symmetry and rotational symmetry

### Exercise

Use the graph above to answer these questions.

1. Which two other angles have the same sine as  $\sin 0^\circ$  ?
2. Between  $0^\circ$  and  $360^\circ$  which one other angle has the same sine as  $\sin 30^\circ$  ?
3. Which two angles have a sine which is the negative of  $\sin 30^\circ$  ?
4. Which angle has the same cosine as  $\cos 0^\circ$  ?
5. Between  $0^\circ$  and  $360^\circ$  which one other angle has the same cosine as  $\cos 10^\circ$  ?
6. Between  $0^\circ$  and  $360^\circ$  which one other angle has the same tangent as  $\tan 20^\circ$  ?

## Trigonometric Equations



Look at the dotted lines on the graph.

If  $\sin x^\circ = -0.5$  check that you can see  $x = 210^\circ$  and  $x = 330^\circ$ .

We have solved a trigonometric equation. However we must say that we have only found answers between  $0^\circ$  and  $360^\circ$ . We can write this as:  $0^\circ \leq x \leq 360^\circ$ .

This is how a question would be:

Solve the equation  $\sin x^\circ = -0.5$  in the range  $0^\circ \leq x \leq 360^\circ$ .

Answer:  $x = 210^\circ$  and  $x = 330^\circ$ .

### Exercise

1. Solve the equation  $\sin x^\circ = 1$  in the range  $0^\circ \leq x \leq 360^\circ$ .
2. Solve the equation  $\sin x^\circ = 0$  in the range  $0^\circ \leq x \leq 360^\circ$ .
3. Solve the equation  $\cos x^\circ = 1$  in the range  $0^\circ \leq x \leq 360^\circ$ . (Look back at the graph on the other side)
4. Solve the equation  $\cos x^\circ = 0.5$  in the range  $0^\circ \leq x \leq 360^\circ$ .
5. Solve the equation  $\sin x^\circ = 0.7$  in the range  $0^\circ \leq x \leq 360^\circ$ . (Use your calculator to find  $\sin^{-1}0.7$ , then use this value and the graph to find the second solution).
6. Solve the equation  $\sin x^\circ = 0.2$  in the range  $0^\circ \leq x \leq 360^\circ$ .
7. Solve the equation  $\sin x^\circ = -0.1$  in the range  $0^\circ \leq x \leq 360^\circ$ .
8. Solve the equation  $\cos x^\circ = 0.8$  in the range  $0^\circ \leq x \leq 360^\circ$ .

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## Worksheet A10: Transpositions of Graphs

**You will need:** graphical calculator or graphing software.

*Remember:*

The greatest least values of sin and cos are 1 and -1

The graphs of sin and cos repeat every  $360^\circ$ .

(We say their **period** is  $360^\circ$ )

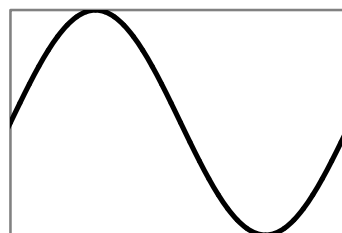
The period of the graph of tan is  $180^\circ$

### Activity

Your aim is to be able to make a quick sketch of the graph of a function such as  $2\sin(x + 30^\circ)$  without using software or a calculator.

Before you start practice drawing a quick sketch of the graphs of sin, cos and tan. You must make sure that you get the points where the graph crosses either axis in the correct place. You must also make sure you show clearly that greatest and least values.

This is fine for sin:



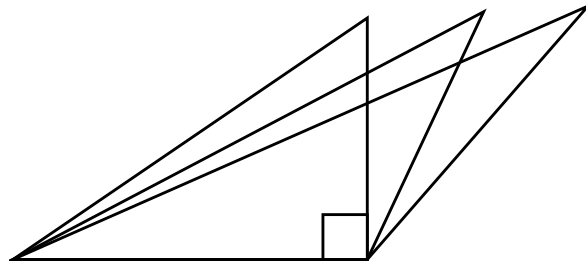
Now use your calculator to draw these graphs and sketch the results (after each set, write down your observations):

- $\sin 2x$ ,  $\sin 3x$ ,  $\sin 4x$
- $2\sin x$ ,  $3\sin x$ ,  $4\sin x$
- $\sin(x + 30^\circ)$ ,  $\sin(x + 50^\circ)$ ,  $\sin(x + 90^\circ)$
- $\sin(x - 30^\circ)$ ,  $\sin(x - 70^\circ)$ ,  $\sin(x - 90^\circ)$
- $3\sin(x + 20^\circ)$ ,  $\sin(3x - 90^\circ)$ ,  $4\sin(x - 60^\circ)$
- $\cos 2x$ ,  $2\cos(x + 20^\circ)$ ,  $2\cos 3x$

Now sketch these without using your calculator:

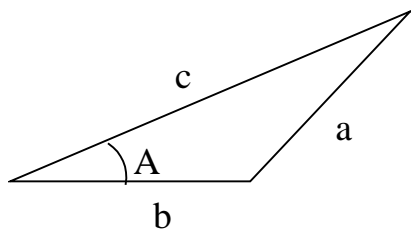
- $\sin 5x$ ,  $\cos(x - 50^\circ)$ ,  $2\sin(x + 30^\circ)$

## Worksheet A11: The Cosine Rule



When the triangle is right angled, we can use Pythagoras theorem to find the hypotenuse.

When the angle is greater (or less) than  $90^\circ$ , we need to extend Pythagoras theorem.



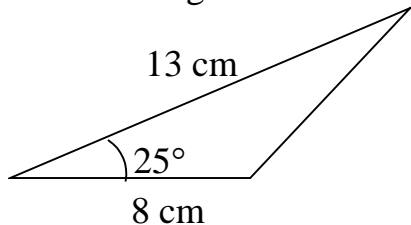
The Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Note:* Side **a** is opposite to angle **A**

### Example

Find the length of the missing side.



*Solution*

Call the missing side **a**.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 25^\circ \\ &= 44.49 \end{aligned}$$

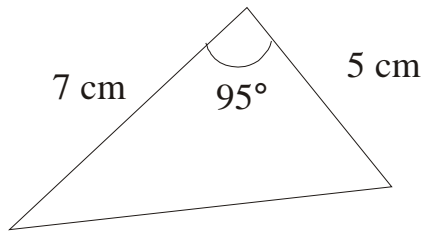
(Using a calculator)

$$\underline{a = 6.7 \text{ cm}}$$

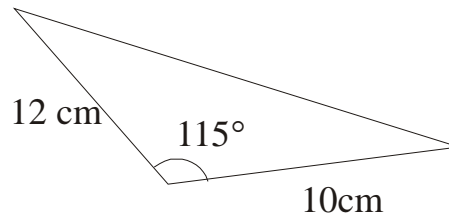
## Exercise

Find the missing side in each of these triangles.

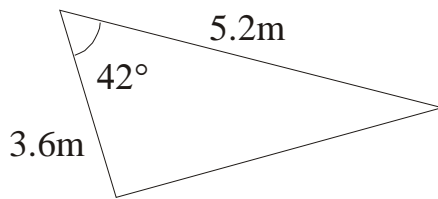
1.



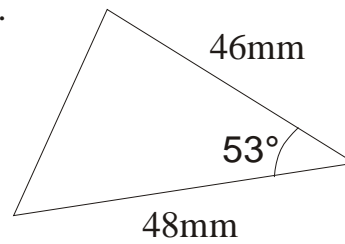
2.



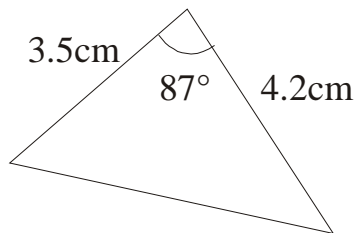
3.



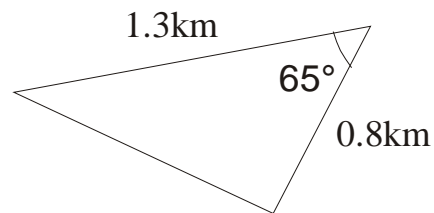
4.



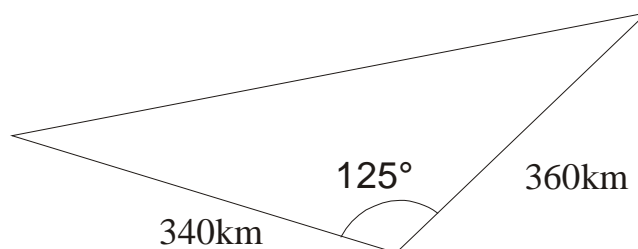
5.



6.



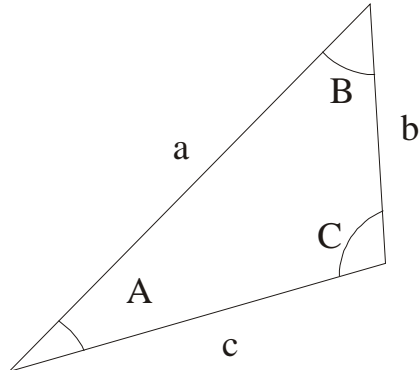
7. An aircraft is forced to detour. It flies 340 km off course then turns through  $125^\circ$  and flies a further 360 km.  
Find the direct distance it would normally have flown.



## Worksheet A12: The Sine Rule

The cosine rule is most convenient for calculating the third side where two sides and an angle are known.

Where two angles are known, or an angle is needed and two sides and an angle are known, we need an alternative rule.

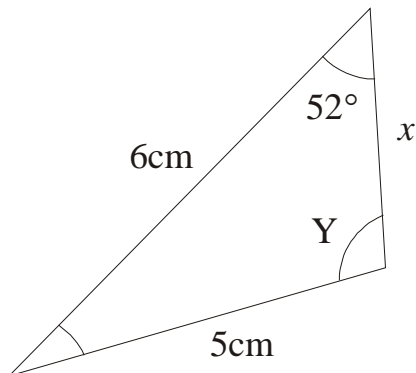


### The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Example

Find the size of the side  $x$  and the angle  $Y$ .



### Solution

To find  $Y$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{5}{\sin 52} &= \frac{6}{\sin Y} \\ \sin Y &= 6 \times \frac{\sin 52}{5} \\ \sin Y &= 0.946 \\ Y &= 71^\circ\end{aligned}$$

To find  $x$ ,  $Y = 71^\circ$  so the third angle is  $47^\circ$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{5}{\sin 52} &= \frac{x}{\sin 47} \\ x &= 5 \times \frac{\sin 47}{\sin 52} \\ x &= 4.6\text{cm}\end{aligned}$$

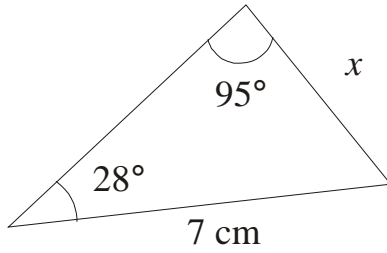




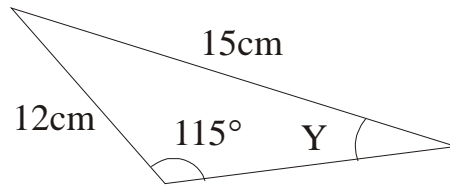
## Exercise

Find the side marked  $x$  or the angle marked  $Y$  in each of these triangles.

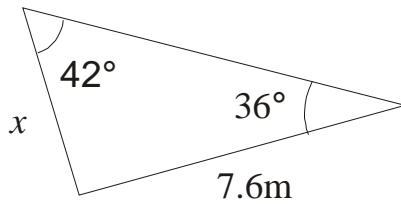
1.



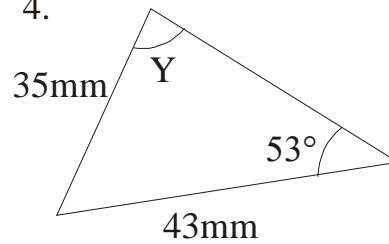
2.



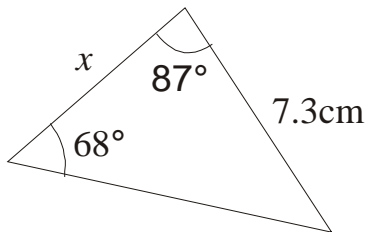
3.



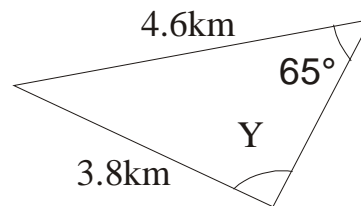
4.



5.



6.



*Hint:* work out the third angle first.

7. A boat is forced to take a detour. Normally it would travel  $28\text{ km}$  due East. Now it has to turn and travel  $7\text{ km}$  before turning again through  $105^\circ$ . Find the bearing of the first leg of the detour and how long is the second leg of the detour.

