Squares in Rectangles: An Investigation

Aims
A two lesson set for year 7 or 8.
How many squares will fit into an \(n \times m\) rectangle?
The teaching strategy follows the aim to develop the following skills:
1. Understanding the problem (experimentation followed by whole class discussion)
2. Having the resources to collect the data (Cut out sheet of different sized squares to fit onto data sheet)
3. Data collection, checking for completeness (Focus: whole class example, then students work in groups with a designated ‘checker’)
4. Tabulating (Prepared data collection sheet)
5. Finding rules (Group discussion followed by whole class round up, section on data collection sheet for groups to say what they have found out)

Lesson 1: Squares on a Chessboard

- Time 1 hour
- The aim of this lesson is for pupils to engage in the investigation skills outlined above while working towards a conclusion to the question: “How many squares on a chessboard”

Starter: (20 mins) – Brainstorm and discussion
Students should be arranged in groups of 3. They will have assigned roles for the activity section.
Show students a chessboard
Brainstorm: ‘How many squares on a chessboard’.
Whole class discussion to develop appropriate terminology to say for example: “How many 1-squares are there in an 8 \(\times\) 8 square”.

Activity: (30 mins)
Students must assign roles within their group and write these in the spaces on the worksheet. (1 scribe, 1 counter, 1 checker). Emphasise the need to check before entering information.
Each group draws an 8 by 8 square on their squared paper. They cut out a set of smaller squares 1 \(\times\) 1, 2 \(\times\) 2, 3 \(\times\) 3 etc.
After pupils have practiced moving their squares to count up the total number of squares possible with each size: Give out worksheet.
Students should all fill in the what I have found out section .... before ....

Summary: (10 mins)
Groups suggest their conclusions, which are summarised on the board.
Lesson 2: Squares in Rectangles

- Time 1 hour
- The aim of this lesson is for pupils to engage in the investigation skills outlined above while working towards a conclusion to the question: “How many squares will fit into any size of rectangle”

**Starter:** (20 mins) – Brainstorm and discussion

Students remain in the same groups of 3. Show students a chessboard
Brainstorm: ‘How many squares will fit into any size of rectangle’.
Whole class discussion:
How can we use the conclusion from the chessboard situation? What are the differences? What are the similarities?
What additional terminology do we need? (You will need to refer to for example a 2\times3 rectangle).

**Activity:** (30 mins)
Pupils can take squared paper and scissors to draw rectangles and re-use there cut-out squares from the last lesson.
Give out worksheet
Work through some rectangles 1 unit wide and a couple of 2 unit wide examples and have students fill in the sheet. Students must have assigned roles, but different people to take different roles this time.
Students should all fill in the what I have found out section .... before ....

**Summary:** (10 mins)
Groups suggest their conclusions, which are summarised on the board.
How Many Squares on a Chessboard?

1 by 1          = Total:

2 by 2         = Total:

3 by 3  = Total:

4 by 4  = Total:

Continue on squared paper.

Fill in the table:

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<th>1 by 1</th>
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<th>5 by 5</th>
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This is what we have found out:
Squares in Rectangles

Write your results in these tables

Rectangles which are 1 square wide

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This is our rule for rectangles which are 1 square wide:

Rectangles which are 2 squares wide

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This is our rule for rectangles which are 2 squares wide:

Rectangles which are 3 squares wide

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This is our rule for rectangles which are 3 squares wide:

Continue on squared paper

This is what we have found out: